Electoral Competition and Party Positioning\textsuperscript{1}

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Abstract

We survey the literature on the policy positioning of political parties in uni- and multi-dimensional policy spaces. We keep throughout the survey the assumption that two parties compete for elections and commit to implementing their policy proposal once elected. The survey stresses the importance of three modeling assumptions: (i) the source of uncertainty in election results, (ii) parties’ objectives (electoral – maximizing their expected vote share or their probability of winning the elections– policy oriented or both), and (iii) voters’ preferences (if and how they care for parties beyond the policies implemented by the winner).
1 Introduction

The canonical model of electoral competition is due to Hotelling (1929) and Downs (1957). They assume that two parties (or candidates) simultaneously propose uni-dimensional policy platforms to voters and commit to implementing their platform if elected. In addition, parties are assumed to care only about winning the election and voters only about policies. Moreover, parties perfectly anticipate the election outcome when choosing their platforms.

As stressed by Roemer (2006), there are many problems associated with this approach and with the results it generates. Assuming that parties do not care for policies contradicts how they have developed historically. These models leave no room for voters to care about the identity of the elected party (as opposed to the policy the party enacts). The lack of electoral uncertainty at the platform selection stage is a rather strong assumption. With regards to the results of the model, the convergence of parties to the same policies is not observed in practice, and this model has generically no equilibrium in pure strategies as soon as we move to a multi-dimensional policy space.

The recent literature on the positioning of political parties has addressed all these issues, and our objective in this paper is to survey the most important contributions to this field. Due to length constraints, we maintain throughout the survey the assumption that two parties compete by committing to implement their policy proposals once elected. We stress the importance of three modeling assumptions: (i) the source of electoral uncertainty, (ii) parties’ objectives (electoral–maximizing their expected vote share or their probability of winning the election–or policy oriented or both), and (iii) voters’ preferences (if and how they care for parties beyond the policies implemented by the winner).

We describe the general setting and notation used throughout the survey in Section 2. Section 3 studies the case where parties have a “pure” objective (elec-
toral or policy oriented), while Section 4 analyzes settings where parties have mixed objectives, either because they care both about policy and about winning, or because they are composed of factions. We study, in Section 5, the case where all voters agree that one party is more attractive than another on non-policy dimensions, and we analyze how this so-called valence advantage impacts the electoral equilibrium. Section 6 concludes with a selection of a few messages we draw from his survey.

2 General setting and notation

This Section describes the basic setting and notation used throughout the survey. The $d$-dimensional policy space $X$ is a non-empty, convex and compact subset of the Euclidean space.\footnote{In order to keep a unified framework, we will mostly refrain from stating how the results and insights presented here should be amended (or not) if we assumed instead that $X$ were a finite set.} There is a finite and odd number of voters, $N = \{1, 2, ..., n\}$.

There are two political parties or candidates (we use the two terms interchangeably throughout the survey), denoted $A$ and $B$. Prior to the election, each party chooses a policy platform, denoted respectively by $x_A$ and $x_B$ in $X$, committing to implement this platform if elected. Parties make simultaneous choices. After observing the announced policy platforms, voters select which party to vote for, with the party garnering the most votes winning the election.

From the perspective of the voters, there are two possible electoral situations: either party $A$ wins and the utility of voter $i$ is given by $U_i(\theta_i, A, x_A)$ or party $B$ wins and $i$’s utility is $U_i(\theta_i, B, x_B)$.\footnote{We rule out a tie because we don’t allow for abstention and assume an odd number of voters.} In words, voters may care both for the enacted policy and for the identity of the winning party. The first argument of $i$’s utility, $\theta_i$, identifies voter $i$’s type.\footnote{Introducing both an index $i$ in the utility function $U$ and a type $\theta_i$ is redundant (since the type space can be multi-dimensional) but proves useful in Section 3.2 when $\theta_i$ is a scalar.}
Faced with only two options, the behavior of the voters is pretty straightforward: Voter $i \in N$ votes for $A$ if $U_i(\theta_i, A, x_A) > U_i(\theta_i, B, x_B)$ and votes for $B$ if $U_i(\theta_i, A, x_A) < U_i(\theta_i, B, x_B)$. When $U_i(\theta_i, A, x_A) = U_i(\theta_i, B, x_B)$, we assume $i$ flips a fair coin to decide between the two parties. The voting stage is simple with two parties since there is no room for strategic participation nor for strategic voting (See Blais and Degan (2017) and Laslier and Nunez (2017) in this special issue on that question).

Let $V_i$ denote the vote of $i$ with the convention that $V_i = 1$ if $i$ votes for $A$ and $V_i = 0$ if $i$ votes for $B$. Given a profile of votes $V = (V_1, \ldots, V_n)$, we denote $\text{Maj}(V)$ the majority outcome in favor of party $A$ as

$$\text{Maj}(V) = \begin{cases} 
1 & \text{if } \sum_{i \in N} V_i > \frac{n}{2} \\
0 & \text{otherwise.}
\end{cases}$$

The utilities of parties $A$ and $B$ for the outcome $(x_A, x_B, V)$ are denoted respectively $U_A(x_A, x_B, V)$ and $U_B(x_A, x_B, V)$. This formulation is flexible as it allows for parties to care about electoral outcomes and/or about the policies proposed by both parties and/or implemented after the vote. More precisely, following Duggan (2014), we consider three types of “pure” motivations for the parties, including two electoral ones. Parties are *Win Motivated* if they only care about winning

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4This randomization is not in contradiction with rationality but removes one degree of freedom when looking for equilibrium as forcefully demonstrated by Duggan and Jackson (2005).

5The question of equivalence between different electoral objectives, first seriously studied in the 1970s (Aranson *et al.* 1974; Hinich 1977; Ledyard 1984), has been the subject of renewed interest recently (Duggan 2006; Patty 2001, 2007). We follow the literature and we do not provide motivations for these assumptions, but rather study their electoral consequences. Note that Aranson *et al.* (1974) contains an interesting discussion of how the constituency characteristics may influence the nature of a candidate’s information, which in turn may influence the candidate’s electoral objective (so that candidates for minor offices may have objectives which differ from those of candidates running for more important offices).
the election, in which case their utilities are given by

\[ U_A(x_A, x_B, V) = \text{Maj}(V) \quad \text{and} \quad U_B(x_A, x_B, V) = 1 - \text{Maj}(V). \]  

(1)

Parties are \textit{Vote Motivated} if they care about the size of their electoral support:

\[ U_A(x_A, x_B, V) = \sum_{i \in N} V_i \quad \text{and} \quad U_B(x_A, x_B, V) = n - \sum_{i \in N} V_i. \]  

(2)

Alternatively, parties can be motivated by the policy implemented following the vote. In this case, we assume that parties preferences for policies are described by differentiable and strictly concave utility functions \( u_A \) and \( u_B \) defined over \( X \) with their most-preferred policies given by \( \tilde{x}_A \) and \( \tilde{x}_B \). The preferences of parties with \textit{Policy Motivation} can be represented as

\[
U_A(x_A, x_B, V) = \text{Maj}(V) u_A(x_A) + [1 - \text{Maj}(V)] u_A(x_B), \\
U_B(x_A, x_B, V) = \text{Maj}(V) u_B(x_A) + [1 - \text{Maj}(V)] u_B(x_B).
\]

We are looking for Nash equilibria in pure strategies of the policy positioning game played by parties. In the absence of such an equilibrium, we briefly say a few words concerning Nash equilibria in mixed strategies.

We now present the results obtained when parties have “pure” motivations (i.e., when they exhibit either a Win, Vote or Policy motivation).

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\( ^6 \)As argued by Roemer (2001) and Grossman and Helpman (2001), parties may have preferences over policies because they represent different ideological (i.e., policy motivated) interest groups in society. Austen-Smith and Banks (2005, p. 255) stress an important difference between electoral and policy motivations: while the former are symmetrical across candidates (as is clear from the objectives (1) and (2)), the latter introduce asymmetries because parties differ in their evaluation of policies, as represented by their policy utility functions \( u_j(x) \).
3 Pure motives

Section 3.1 examines the results of the deterministic case where parties perfectly anticipate the voting behavior $V_i$ of all voters $i \in N$ at the policy platform selection stage. We then allow parties to be uncertain as to the electoral consequences of their choice of platform. Section 3.2 studies the case where parties are uncertain as to voters’ preferences for parties (independently of their policy preferences), while Section 3.3 presents results when parties are uncertain as to voters’ preferences over policies. In both sections, we assume that parties maximize their expected utility given their beliefs about voters’ preferences.\(^7\)

### 3.1 Deterministic case

In this section, we assume away uncertainty, so that parties know voters’ preferences and can compute with certainty the voting behavior of all individuals when choosing their platforms. Moreover, we assume that voters care only about implemented policies, so that, voter $i$’s utility is given by

$$U_i(\theta_i, j, x_j) = u_i(x_j) \quad \text{for} \quad j \in \{A, B\},$$

where $u_i$ is differentiable and strictly concave in $X$ and $\bar{x}_i$ denotes $i$’s ideal policy.\(^8\)

It is well known since Hotelling (1929) and Downs (1957) that,

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\(^7\)This section is based on the survey by Duggan (2014).

\(^8\)Concavity is a strong assumption, as stated forcefully by Osborne (1995): “The assumption of concavity is often adopted, first because it is associated with ‘risk aversion’ and second because it makes it easier to show that an equilibrium exists. However, I am uncomfortable with the implication of concavity that extremists are highly sensitive to differences between moderate candidates (a view that seems to be shared by Downs 1957, 119-20). (...) Further, it is not clear that evidence that people are risk averse in economic decision-making has any relevance here.” We nevertheless maintain this assumption throughout the survey, since it is done by most papers in the literature. For an exception to this rule, see footnote 15.
Proposition 1 Assume that there is no uncertainty, that $d = 1$ and that both parties are electorally motivated (i.e., they have either a Win or a Vote Motivation), then there exists a unique Nash equilibrium in pure strategies where both parties propose the median voter’s ideal point: $\text{med}(\tilde{x}_1, \ldots, \tilde{x}_n)$.

This result is known as the “median voter theorem.” Moreover, it is also known since Plott (1967) and Hinich et al (1973), among others,\(^9\) that

Proposition 2 Assume that there is no uncertainty, that $d > 1$ and that both parties are electorally motivated (i.e., they have either a Win or a Vote Motivation), then there generically does not exist any Nash equilibrium in pure strategies.

The generalization of Proposition 1 to a multidimensional setting requires the existence of a “median in all dimensions” of the policy space. This in turn requires that the distribution of voters’ most-preferred policies be radially symmetric, which is an extremely restrictive assumption. Moreover, any move from a radially symmetric distribution of blisspoints, however small, results in the (generic) non-existence of an equilibrium in pure strategies.

As for equilibria in mixed strategies, there is no general existence theorem because the parties’ payoffs are not continuous, in general, when $d > 1$. Duggan and Jackson (2005) show that, if indifferent voters are allowed to randomize with any probability between zero and one (rather than with 1/2 as assumed here), then mixed strategy equilibria do exist. Moreover, they show that, starting from a distribution of individuals such that a pure strategy Nash equilibrium exists, and perturbing this distribution, the equilibrium mixed strategies of the parties put probabilities arbitrarily close to one on policies near the original pure strategy Nash equilibrium. In other words, mixed strategy equilibrium outcomes change in

a continuous way when voter preferences are perturbed.\textsuperscript{10} Also, mixed equilibria differ according to whether the parties are win or vote motivated.\textsuperscript{11}

What about moving away from electoral preferences towards policy motivations? Unfortunately, this does not change the results with $d = 1$, as shown in the next proposition due to Wittman (1977), Calvert (1985) and Roemer (1994):

**Proposition 3** Assume that there is no uncertainty, that $d = 1$ and that both parties are Policy motivated with $\bar{x}_A < \text{med}(\bar{x}_1, ..., \bar{x}_n) < \bar{x}_B$, then there exists a unique Nash equilibrium in pure strategies where both parties propose the median voter’s most-preferred blisspoint: $\text{med}(\bar{x}_1, ..., \bar{x}_n)$.

Since parties care for the implemented policy, they first have to win the election in order to influence this policy. When parties’ ideal points are on opposite sides of the median voter’s blisspoint, they converge to the median’s ideal point.\textsuperscript{12} Under policy motivation we obtain the same prediction as in the Downsian model where parties are office motivated and compete on a single dimension.

As for multidimensional policy spaces, there seems to be more hope for existence of an equilibrium in pure strategies than under electoral motivations, for the following reason. With electoral motivation, a party would find it profitable to deviate to any policy which is preferred by a majority to the policy proposed by its opponent. With policy motivations, a profitable deviation must moreover increase the utility of the deviating party. There are then fewer potentially profitable deviations.

\textsuperscript{10}McKelvey (1986), Cox (1987), and Banks, Duggan and Le Breton (2002) show that the support of the mixed strategy Nash equilibria lies in the uncovered set, a centrally located subset of the policy space.

\textsuperscript{11}More precisely, in the case where $X$ is finite, Laffond, Laslier and Le Breton (1994) exhibit a profile of preferences such that the win motivation and vote motivation games (generated from this profile) have both a unique Nash equilibrium in mixed strategies with totally disjoint supports.

\textsuperscript{12}If $\text{med}(\bar{x}_1, ..., \bar{x}_n) < \bar{x}_A < \bar{x}_B$, then both parties proposing $\bar{x}_A$ is an equilibrium, since $A$ gets its most-preferred policy, while $B$ can only affect the implemented policy by proposing a policy to the left of $\bar{x}_A$, which it dislikes even more than $\bar{x}_A$. 

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Unfortunately, Duggan and Fey (2005) prove that an equilibrium in pure strategies will almost never exist when \( d > 2 \). More precisely, they come up with Plott-like conditions where voters with exactly opposite preferences in the \( d \)-dimensional space are paired with each other. Interestingly, in the knife edge case where such an equilibrium exists when \( d \geq 2 \), both Duggan and Fey (2005) and Roemer (2001) show that a near universal feature of the equilibrium is that both parties propose the same policy, which is most-preferred by at least one voter. Finally, the results by Duggan and Jackson (2005) apply here as well, so that a mixed equilibrium exists if indifferent voters randomize in a flexible way.

The message of this section is thus pretty bleak: if \( d = 1 \), we obtain convergence to the median blisspoint irrespective of the (electoral or policy) motivation of parties, while there is generically no equilibrium in pure strategies with \( d > 2 \).

We now introduce uncertainty, so that parties see individual voters’ behavior as random. We first look at situations with stochastic partisanship, then to those with stochastic preferences.

### 3.2 The stochastic partisanship approach

In this section, voters care both for implemented policies and for the identity of the winning party, with parties, at the campaign selection stage, being uncertain as to voters’ preferences and thus on how they will vote. In the **stochastic partisanship model** parties see voters’ preferences as affected by a random shock determining the bias the voter has for one of the two parties. More precisely, voters have additively separable preferences over policies and biases, so that

\[
U_i(\theta_i, A, x_A) = u_i(x_A) \quad \text{and} \quad U_i(\theta_i, B, x_B) = u_i(x_B) + \theta_i,
\]
where \( u_i(. \mid \theta) \) is differentiable and strictly concave in \( X \) and where the vector of party biases \((\theta_1, \ldots, \theta_n)\) is seen as the realization of a random variable by both candidates.\textsuperscript{13} More precisely, both parties have the same beliefs about voters, assuming that each \( \theta_i \) is distributed according to the differentiable distribution function \( F_i \) with pdf \( f_i > 0 \). Observe that biases are not necessarily independently distributed. The probability that voter \( i \) votes for \( A \), which we denote by \( P_i(x_A, x_B) \), is then given by the probability that his bias \( \theta_i \) for \( B \) (which can be negative) is lower than the difference in utility \( u_i(x) \) between \( A \)'s and \( B \)'s policy proposals, \( \theta_i < u_i(x_A) - u_i(x_B) \), so that,

\[
P_i(x_A, x_B) = F_i(u_i(x_A) - u_i(x_B)).
\] (3)

We will only study the Vote motivation for parties in this section and refer the reader to Duggan (2014) for win and policy motivations (which have been less extensively studied in the literature). Party \( A \) chooses \( x_A \) to maximize \( \sum_{i \in N} P_i(x_A, x_B) \) while party \( B \) chooses \( x_B \) to maximize \( n - \sum_{i \in N} P_i(x_A, x_B) \).

The next proposition has been proven by Hinich (1977, 1978), Lindbeck and Weibull (1987, 1993) and has been generalized by Banks and Duggan (2005):

\textbf{Proposition 4} In the stochastic partisanship model with Vote motivation and \( d \geq 1 \), if \( (x_A^*, x_B^*) \) is an interior equilibrium in pure strategies, then

\[
x_A^* = x_B^* = \bar{x} = \arg \max_{x \in X} \sum_i f_i(0)u_i(x).
\] (4)

In words, both parties converge to the same policy \( \bar{x} \), which is the (unique) policy maximizing the weighted sum of the voters’ utilities, where the weights are

\textsuperscript{13}Coughlin and Nitzan (1981) study the multiplicative formulation of this problem, where voter \( i \)'s utility function is log-concave with \( i \) voting for party \( A \) if \( u_i(x_A) \geq u_i(x_B)\theta_i \). Duggan (2014) indeed shows that the two approaches are equivalent, up to a simple transformation.
given by those that correspond to the densities of the voters’ biases at zero, \( f_i(0) \).

The intuition for this result is that the “neutral” voters (those with \( \theta_i = 0 \)) are the ones whose votes are the most easily swayed in favor of the party. As both parties compete to attract these swayable voters, they end up proposing the same platform.\(^{14}\) Proposition 4 holds whatever the dimensionality of the policy space.

Proposition 4 does not tackle the problem of the existence of the equilibrium. Observe from (3) that the probability that a given individual \( i \) votes for \( A \) is a continuous function of \( A \)’s proposal. This translates into the continuity of the expected vote function with respect to the party’s proposal. In other words, introducing uncertainty smooths the parties’ objectives. The other condition (beyond continuity) needed to have an equilibrium in pure strategies is that parties’ objectives be quasi-concave. The following proposition (due to Hinich, Ledyard and Ordeshook, 1972, 1973; Enelow and Hinich, 1989; and Lindbeck and Weibull, 1993) gives sufficient conditions on the distribution of voters’ biases – i.e., on the distribution function \( F_i \) – to have an equilibrium:

**Proposition 5** In the stochastic partisanship model with vote motivation and \( d \geq 1 \), sufficient conditions for the existence of an equilibrium in pure strategies are that (i) \( F_i(u_i(x) - u_i(y)) \) is concave in \( x \) and (ii) \( F_i(u_i(y) - u_i(x)) \) is convex in \( x \), for each voter \( i \) and all policies \( y \in X \).

We refer the reader to Duggan (2014, p21-22) for an intuitive geometric discussion of this proposition.

Propositions 1 and 4 may seem to clash with each other, in the following sense: without uncertainty and with \( d = 1 \), the unique equilibrium in pure strategies is for both parties to propose the median’s ideal point. Adding a small amount

\(^{14}\)The policy \( \bar{x} \) has no particular normative appeal, since there is no normative general reason for any social planner to use these specific weights. A special case arises where all voters \( i \) share the same distribution \( F_i \), in which case policy \( \bar{x} \) is the utilitarian optimum (maximizing the unweighted sum of utilities).
of uncertainty (with distributions functions $F_i$ of biases converging to the point mass on zero) then moves the equilibrium policy to (the utilitarian–unweighted–optimum) $\bar{x}$. This apparent clash can be explained away thanks to Laussel and Le Breton (2002) and Banks and Duggan (2005) who proved that the equilibrium in pure strategies fails to exist in the stochastic partisanship model when voting behavior is close to deterministic. In other words, one needs sufficiently large uncertainty for the sufficient conditions mentioned in Proposition 5 to hold. It is worth stressing this point, since it indicates that stochastic partisanship probabilistic voting can actually create existence problems in one-dimensional settings where a deterministic, Downsian equilibrium in pure strategies exists.

Thanks to the continuity of parties payoffs, an equilibrium in mixed strategies exists in stochastic partisanship probabilistic models. Moreover, Banks and Duggan (2005) show that the support of this equilibrium converges to the median’s most-preferred policy when the amount of noise goes to zero. Finally, observe that moving to a win motivation for parties makes the existence problem worse, in the sense that the conditions enunciated in Proposition 5 are not sufficient anymore to guarantee existence of a Nash equilibrium in pure strategies (see Duggan, 2014, Section 4.2, and Patty, 2005). In other words, it is even more difficult to generate quasi-concave payoff functions for parties with a win motivation, compared to a vote motivation.\footnote{Kamada and Kojima (2014) shows that under win motivation and enough convexity of the utility function, stochastic partisanship produces divergence at equilibrium.}

### 3.3 The stochastic preferences approach

An alternative way to introduce uncertainty is to consider that parties do not know voters’ policy preferences (as opposed to their partisan bias). We assume that voter $i$’s preferences for party $j$, for all $i \in N$, is represented by

\[\text{\ldots}\]
\[ U_i(\theta_i, j, x_j) = u_i(\theta_i, x_j) \quad \text{for} \quad j \in \{A, B\}, \]

where \( \theta_i \in \Theta \) is a preference parameter in the set \( \Theta \). Voter \( i \)'s utility function, \( u_i(\theta_i, x) \), is differentiable and concave in its second argument with ideal policy \( \bar{x}_i(\theta_i) \). The uncertainty of parties \( A \) and \( B \) regarding the distribution of voters’ policy preferences is described by the distribution function \( G_i, \ i \in N \).\textsuperscript{16} Thus, both parties believe that the probability that \( i \) votes for \( A \) when each party \( j \) proposes \( x_j \) is

\[
P_i(x_A, x_B) = G_i(\theta_i \in \Theta : u_i(\theta_i, x_A) > u_i(\theta_i, x_B)) + \frac{1}{2} G_i(\theta_i \in \Theta : u_i(\theta_i, x_A) = u_i(\theta_i, x_B)).
\]

Observe that we need not assume that the types \( \theta_i \) are drawn independently, but rather that the types are sufficiently dispersed, in which case the second term of \((5)\) disappears (see Duggan, 2014, Section 5 for the full mathematical statement).

We denote by \( H_i \) the distribution of voter \( i \)'s ideal policy induced by \( G_i \). In the unidimensional case \((d = 1)\), \( H_i(x) \) denotes the probability that \( i \)'s ideal policy is less than or equal to \( x \). When preferences are quadratic, the probability that \( i \) votes for \( A \) when \( x_A < x_B \) simplifies to

\[
P_i(x_A, x_B) = H_i \left( \frac{x_A + x_B}{2} \right),
\]

and the probability that \( A \) wins is defined by

\[
P(x_A, x_B) = H_\mu \left( \frac{x_A + x_B}{2} \right),
\]

\textsuperscript{16}If voters are ex-ante identical for the two parties, the subscript \( i \) can be dropped from \( u \) and \( G \) assuming without loss of generality that there is a single voter. See the results in Duggan (2014) pertaining to this situation, termed the \textit{representative voter stochastic preference model.}
where $H_\mu$ denotes the distribution of the median voter’s ideal policy.

We begin with the vote motivation where $A$ and $B$ respectively maximize $\sum_{i \in N} P_i(x_A, x_B)$ and $n - \sum_{i \in N} P_i(x_A, x_B)$. Define $H_\alpha(x)$ as the average distribution of voters’ ideal policies and $x_\alpha$ its median, so that,

$$H_\alpha(x) = \frac{1}{n} \sum_{i \in N} H_i(x).$$

Duggan (2006) proves the following Proposition.

**Proposition 6** In the stochastic preference model with vote motivation and $d = 1$, there is a unique equilibrium in pure strategies $(x^*_A, x^*_B)$ in which both candidates locate at the median of the average distribution: $x^*_A = x^*_B = x_\alpha$.

In the win motivation, $A$ and $B$ maximize $P(x_A, x_B)$ and $1 - P(x_A, x_B)$. Let $x_\mu$ denote the median of the $H_\mu$ distribution. Calvert (1985) shows that

**Proposition 7** In the stochastic preference model with win motivation and $d = 1$, there is a unique equilibrium in pure strategies $(x^*_A, x^*_B)$ in which both candidates locate at the median of the distribution of median policies: $x^*_A = x^*_B = x_\mu$.

We can see that the type of electoral motivation matters here –in stark contrast to the deterministic case– since parties converge to the average median with vote motivation (Proposition 6), and to the median of medians under win motivation (Proposition 7). In both cases, their electoral motivation (whether win or vote) pushes both parties to converge to the median of some distribution of most-preferred points. Under vote motivation, parties maximize the sum of the probabilities that individual agents vote for them, and thus care about the average distribution of voters’ ideal policies. When parties care only about winning they care about the distribution of median’s ideal policies.
We turn to the more developed analysis of this model under policy motivations, where $A$ chooses $x_A$ to maximize

$$P(x_A, x_B)u_A(x_A) + [1 - P(x_A, x_B)] u_A(x_B),$$

while $B$ chooses $x_B$ to maximize

$$P(x_A, x_B)u_B(x_A) + [1 - P(x_A, x_B)] u_B(x_B),$$

assuming that their utility functions $u_j(x), j \in \{A, B\}$, are concave over $X$. A Nash equilibrium in pure strategies of this game is dubbed a Wittman equilibrium. The following proposition has been proven in various guises by Wittman (1983, 1990), Hansson and Stuart (1984), Calvert (1985) and Roemer (1994):

**Proposition 8** In the stochastic preference model with policy motivations and $d \geq 1$, if $(x_A^*, x_B^*)$ is an equilibrium in pure strategies (or Wittman equilibrium), then the candidates do not locate at the same policy position: $x_A^* \neq x_B^*$.

When selecting their platforms, parties face a trade-off between increasing their probability of winning and moving closer to their ideal policy. Since parties have different policy preferences, they end up proposing different platforms. For instance, for $d = 1$, quadratic utilities with parties’ ideal policies being such that $\tilde{x}_A < \tilde{x}_B$, the equilibrium $(x_A^*, x_B^*)$, if it exists, is of the form $\tilde{x}_A < x_A^* < x_B^* < \tilde{x}_B$.

Calvert (1985) and Roemer (1994) further show that, if the policy space is unidimensional ($d = 1$), the stochastic preferences model with policy motivation gets close to Downsian model, in the sense that the equilibrium policies (assuming equilibrium existence) of both candidates converge to the median’s ideal policy as the amount of noise added to the Downsian model goes to zero.

Observe that, unlike in the stochastic partisanship approach, both the proba-
bility of winning and the vote share functions are discontinuous along the diagonal—
i.e., when a party crosses over the other one by proposing the same policy. As
for equilibrium existence with policy motivation, the good news is that the dis-
continuity of the probability of winning function when both parties propose the
same policy does not translate into a discontinuous payoff function. The intuition
is that the discontinuity in $P(x_A, x_B)$ occurs when both parties propose the same
policy, so that the utility obtained by a party is anyway the same with both poli-
cies. On the other hand, quasi-concavity of the payoff function is not guaranteed,
so one needs additional assumptions for equilibrium existence. These assumptions
are not very strict in the case of a unidimensional policy space. For instance, when
there is a continuum of voters, Roemer (1997) proves that a sufficient condition for
equilibrium existence if $d = 1$ is that the distribution of the median ideal points
among citizens be log concave. Roemer (2001, p.68) concludes that “we find that
in most interesting examples Wittman equilibria exist, but a truly satisfactory
general existence theorem is not known.”

Sufficient conditions for existence are more difficult to find when $d > 1$ and,
to the best of our knowledge, there is no general proof of existence for multi-
dimensional policy spaces.

Equilibria in mixed strategies exist and are continuous in the Downs model (as
support of any mixed strategies equilibrium converges to the Downsian outcome
when the amount of noise vanishes, see more in Duggan, 2014).

We now study models where parties care about both electoral and policy out-
comes.
4 Mixed motives and Parties as Coalitions

Up to now, we have concentrated on pure motivations, either electoral (win or vote) or ideological (policy). In reality, objectives are probably more complex, with a mix of policy and electoral motivations. We consider two potential approaches to this richer set of objectives. In the first section, we retain the assumption that parties are unitary actors maximizing a single objective, which here takes into account both electoral and policy motives. We then examine how parties determine their positions when they are composed of factions with different motives.

4.1 Mixed electoral and policy motivations

The easiest way to model the fact that parties care both for (enacted) policies and for winning per se is to modify their policy preference (as presented above) to add a spoils of office term that the party gets when winning. We now discuss the analysis from Drouvelis et al (2014), and refer the reader to Duggan (2014) and Saporiti (2008) for other results both in the deterministic and probabilistic voting environments.

Drouvelis et al (2014) examine a special case of the stochastic preferences approach presented above. In their uni-dimensional policy model, $X = [0, 1]$, parties are uncertain as to voters’ preferences and voters only care for the (enacted) policy with voter $i$’s utility decreasing the further party $j$’s policy $x_j$ is from $i$’s ideal policy, $\tilde{x}_i(\theta_i)$, i.e.,

$$U_i(\theta_i, j, x_j) = u_i(\theta_i, x_j) = |x_j - \tilde{x}_i(\theta_i)| \quad \text{for} \quad j \in \{A, B\}.$$ 

Unlike in Section 3.3, no micro-foundations for the parties’ uncertainty about the distribution of $\tilde{x}_i(\theta_i)$ is provided. Parties believe that the median’s ideal point
is uniformly distributed over $[1/2 - \beta, 1/2 + \beta]$ where $\beta > 0$ measures the parties’ degree of uncertainty as to voters’ preferences. As in Section 3.3, let $P(x_A, x_B)$ denote the probability that $A$ wins given the proposed policies $(x_A, x_B)$ where $x_A < x_B$ and the distribution of the median’s ideal policy is given by $H_\mu(\cdot)$. Then

$$P(x_A, x_B) = H_\mu\left(\frac{x_A + x_B}{2}\right) = \begin{cases} 
0 & \text{if } \frac{x_A + x_B}{2} \leq \frac{1}{2} - \beta \\
\frac{1}{2\beta} \left[\frac{x_A + x_B}{2} - \left(\frac{1}{2} - \beta\right)\right] & \text{if } \frac{1}{2} - \beta < \frac{x_A + x_B}{2} < \frac{1}{2} + \beta \\
1 & \text{if } \frac{x_A + x_B}{2} \geq \frac{1}{2} + \beta
\end{cases}$$

Parties care both about enacted policies and about winning per se. Parties’ policy utility functions are the same as that of voters, with

$$u_j(x) = |x - \tilde{x}_j|,$$

and their most-preferred policy options $\tilde{x}_A$ and $\tilde{x}_B$ lying on opposite sides of the political spectrum, so that $\tilde{x}_A < 1/2 < \tilde{x}_B$. Parties care about policies and about winning, with the winning party $j$ also collecting the spoils of office, $\xi_j$. Parties (expected) utility at the proposed policies, $(x_A, x_B)$, are given by

$$U_A(x_A, x_B) = P(x_A, x_B) [u_A(x_A) + \xi_A] + [1 - P(x_A, x_B)] u_A(x_B), \quad \text{and}$$
$$U_B(x_A, x_B) = P(x_A, x_B) u_B(x_A) + [1 - P(x_A, x_B)] [u_B(x_B) + \xi_B].$$

Drouvelis et al (2014) prove the following two propositions.

**Proposition 9** The mixed motivation election game has a pure strategy equilibrium $(x_A^*, x_B^*)$ with $x_A^* = x_B^* = x^*$ if and only if $x^* = 1/2$ and $\xi_j \geq 2\beta$ for all $j = A, B$.

**Proposition 10** The mixed motivation election game has a pure strategy equilibrium $(x_A^*, x_B^*)$ with $x_A^* < 1/2 < x_B^*$ if and only if $\xi_j < 2\beta$ for all $j = A, B$. 

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Proposition 9 shows that both parties converge to the (estimated) median’s ideal point if electoral uncertainty is low for both parties compared with office motivation, while Proposition 10 shows the emergence of two-sided policy differentiation where each party chooses a policy on its ideological side of the estimated median if uncertainty is high enough for both parties.

Proposition 9 is reminiscent of Calvert’s (1985) assertion that small departures from office motivation without uncertainty lead to small departures from convergence. The intuition for the comparison of $\xi_j$ and $2\beta$ can easily be recovered. For party $j$, moving away from its most-preferred policy and towards the center of the policy space has both a marginal utility cost (of one, given the utility function) and a marginal benefit which equals the product of the spoils of office term $\xi_j$ and of the marginal increase in the probability of winning the election. Note that the later is equal to $1/2\beta$, since $2\beta$ is the length of the support of $\theta_m$. Both parties then converge if the marginal benefit is larger than the marginal cost—i.e., if $1 < \xi_j/2\beta$ for both parties $j = A, B$.

Drouvelis et al (2014) also obtain that party proposals become more polarized as $\beta$ increases. This is intuitive, since a higher uncertainty as to the location of the median’s ideal policy decreases the electoral cost (as perceived by the parties) of locating closer to their most-preferred policy.

Finally, Drouvelis et al (2014) study the case where parties have asymmetric motivations (i.e., where $\xi_A \neq \xi_B$). They still obtain convergence to the estimated median when the uncertainty $\beta$ is low enough. When uncertainty increases above a threshold, an equilibrium in pure strategies fails to exists, but they obtain a mixed strategy equilibrium with probabilistic differentiation, where both parties randomize on the same side of the expected median’s policy. As $\beta$ increases further, there is again an equilibrium in pure strategies, where both parties propose different policies. These policies are first located on the same side of the expected
median (one-sided policy differentiation), and then as $\beta$ increases further, on each candidate’s side of the political spectrum (two-sided differentiation).

4.2 Parties and their factions

A strong case can be made that political parties do not behave like unitary actors. They are formed by individuals who may have different motives for being in the party. Party members can be grouped into different factions, who struggle internally when deciding the official line to be taken by the party. Moreover, the rules governing the decision process inside parties may be complex. Roemer (2001, p153-154) contains several historical examples of the struggle between factions inside parties, ranging from the German Social Democratic Party at the beginning of the 20th Century, to the US Republican Party during the 1964 US election.

Roemer (2001) assumes that two factions coexist inside both political parties. In each party, the opportunists care exclusively about winning the elections (and maximize $P(x_A, x_B)$ in party $A$ and $1 - P(x_A, x_B)$ in party $B$), while the militants care about the specific policy proposal $x$ made by their party, and maximize $u_j(x), j = A, B$. There is then a crucial difference between policy motivation (as modelled in the previous sections, where a party cares for the implemented policy) and the behavior of the militants, who care about the policy proposed by their party, rather than for the policy enacted.

Both intra-party factions bargain with each other over the party’s policy platforms. Each faction has a complete preference order on the set of possible policies, and Roemer assumes that the party’s preference ordering is determined by the in-
tersection of these two orders. In other words, unanimity between the two factions is required for a party to accept a deviation from its current policy. This unanimity rule determines the preferences (payoffs) of the two parties who simultaneously choose their political platforms. A party unanimity Nash equilibrium (PUNE) is an equilibrium of this game.\textsuperscript{18}

**Definition 11** The policy pair \((x_A^*, x_B^*)\) is a PUNE if and only if \(\forall (j, k) \in \{A, B\}, j \neq k, \exists x \in X\) such that (i) \(u_j(x) \geq u_j(x_j)\) and (ii) either \(P(x, x_B) \geq P(x_A, x_B)\) or \(1 - P(x_A, x) \geq 1 - P(x_A, x_B)\), with at least one strict inequality.

Roemer does not provide a general existence theorem for the PUNEs, but mentions that PUNEs do exist in all the applications he has studied.\textsuperscript{19} The intuition for why PUNEs exist (even in multidimensional policy spaces) is that the unanimity requirement (between factions) restricts the set of admissible deviations for both parties. Another way to put it is that the unanimity requirement means that each party’s preference ordering over policies is incomplete, since a party can only rank policies if both its factions have the same ordering. Since deviations must fulfill the harsh requirement of pleasing both factions at the same time, the existence of PUNEs in many environments becomes intuitive.

From the definition, it is easy to see that PUNEs constitute an extension of equilibria with (pure) win motivation. Assume that such an equilibrium with win motivation exists. Then, by definition, each party maximizes its probability of winning given the policy choice of the other party. It is then impossible to move away from this policy pair without displeasing the opportunists, so that an equilibrium with win motivation is also a PUNE. We will see shortly that

\textsuperscript{18}Roemer originally assumed the existence of a third faction, the reformists, who exhibit policy motivations as modelled in the previous section, maximizing their (expected) utility from the implemented policy platform. A deviation pleasing both the militants and the opportunists also increases the utility of the reformists, making the latter faction superfluous, since its presence would not modify the characterization of the equilibrium policies.

\textsuperscript{19}See De Donder and Gallego (2017) for a brief survey of PUNE applications.
PUNEs also constitute an extension of equilibria with (pure) policy motivation (i.e., Wittman’s equilibria).

Roemer (2001, Section 8.3) shows that the bargaining that takes place within parties can be represented as a generalized Nash bargaining problem when appropriate convexity properties hold. More precisely, take the threat point of this intra-party bargaining game to be the situation which occurs when the other party wins the election for sure. The Nash bargaining games between militants and opportunists in party $A$ and party $B$ are given by

\[
\max_{x \in X} \left[ P(x, x_B) - 0 \right]^\alpha [u_A(x) - u_A(x_B)]^{1-\alpha}, \quad \text{and} \quad \max_{x \in X} \left[ 1 - P(x_A, x) \right]^\beta [u_B(x) - u_B(x_A)]^{1-\beta},
\]

where $\alpha$ and $\beta$ measure the relative bargaining power of the opportunists in party $A$ and $B$ respectively.

Roemer (2001) establishes that the PUNEs (when they exist) form a two-dimensional manifold whatever the dimensionality of the policy space $d \geq 1$. More precisely, he shows that a PUNE can be expressed as a pair of policies $(x_A, x_B)$ solving equations (6) and (7) simultaneously for some values of $\alpha, \beta \in [0,1]$. The two-dimensional manifold of PUNEs can then be indexed by the pair of bargaining powers $(\alpha, \beta)$. Note that existence of an equilibrium cannot be guaranteed for any pre-specified pair of numbers $\alpha$ and $\beta$. Roemer (2001) shows that the specific case where both factions have the same bargaining power in both parties ($\alpha = \beta = 1/2$) corresponds to the Wittman equilibrium, while the classical Downsian equilibrium (with purely electorally motivated parties) corresponds to $\alpha = \beta = 1$. Also, this result sheds light on why the PUNE concept is generally not useful when $d = 1$: it is insufficiently discriminating in a one dimensional policy space.

This analysis has assumed that parties are exogenous entities, endowed with
a set of militants with well defined preferences. One can go a step further and endogenize the policy preferences of the parties’ militants by assuming that they maximize some aggregate (such as the average) utility of the agents who vote for this party in equilibrium. One then obtains a Party Unanimity Nash Equilibrium with Endogenous Parties (or PUNEEP). We refer the reader to Chapter 13 of Roemer (2001) for several applications of this approach.

In the stochastic partisanship section, Section 3.2, we examined the case where voters disagree on the attractiveness of exogenous, non-policy related characteristics of political parties. The next section studies the case where voters agree that one party has better characteristics than the other.

5 Valence models

Our framework encompasses cases where voters have concerns about candidates that are independent of their policy positions. These concerns may be policies that candidates cannot credibly commit to change (e.g., Grossman and Helpman (2001) distinguish between fixed and pliable policies) but these dimensions may also reflect voters’ evaluations of other characteristics they consider important. In Section 3.2 voters’ views of these characteristics or fixed positions varied, and parties were uncertain as to voters’ individual partisan biases. In this section, we study the setting where voters have similar views on these characteristics (e.g., on competence, corruption, loyalty, charisma…) which we refer to as a valence following the pioneering work of Stokes (1963, 1992).

Stokes argues that while valence issues are exogenously given to candidates at election time, they may vary across candidates, e.g., voters may perceive candidates as differing in ability to govern. If valence were the unique source of heterogeneity besides the ideological heterogeneity captured by the first argument
of the utility function \( U_i(\theta_i, j, x_j), j \in \{A,B\} \), and in the absence of uncertainty, we would obtain that \( U_i(\theta_i, A, x) > U_i(\theta_i, B, x) \) for all \( x \in X \) and all \( i \in N \) if party \( A \) benefits from a valence advantage over party \( B \). Most of the models discussed in this Section can be considered a special case of the stochastic partisanship framework presented in Section 3.2.

We denote party \( j \)'s valence by \( \vartheta_j \), and voter \( i \)'s utility by

\[
U_i(\theta_i, j, x_j) = u_i(\theta_i, x_j) + \vartheta_j \quad \text{for} \quad j \in \{A,B\}.
\]

Assume that \( \vartheta = \vartheta_A - \vartheta_B > 0 \), so that candidate \( A \) has a valence advantage over \( B \). Unlike in Section 3.2, the non-policy parameters \( \vartheta_A \) and \( \vartheta_B \) are common to all voters. Voter \( i \) votes for \( A \) if \( u_i(\theta_i, x_A) - u_i(\theta_i, x_B) + \vartheta > 0 \).

We study first unidimensional policy models then multidimensional models.

5.1 Valence in unidimensional policy spaces

This section shows that models with policy motivated parties where voters care about both policies and valences may generate policy divergence even in the absence of uncertainty (which is not the case without valence, see Section 3.1). For instance, assume that both parties’ and voters’ preferences over (implemented) policies are concave with ideal policies represented by \( \tilde{x}_j \) (\( j = A,B \)) for parties and by \( \tilde{x}_i \) for voter \( i \). Assume further that \( \tilde{x}_A < \tilde{x}_m < \tilde{x}_B \) where \( \tilde{x}_m \) is the median voter’s ideal point, and that party \( A \) wins the election unless \( |x_A - \tilde{x}_m| > |x_B - \tilde{x}_m| + y \), where \( y > 0 \). This means that \( A \) wins if it locates within \( y \) units of the median. When \( \tilde{x}_A < \tilde{x}_m - y \), there is an equilibrium with \( x_B = \tilde{x}_m \) and \( x_A = \tilde{x}_m - y \), so that \( A \) wins for sure and \( B \) prevents \( A \) from moving further to the left.

Candidate uncertainty as to voters’ policy preferences is introduced by Grose-
close (2001) who develops a one dimensional ($d = 1$) mixed motive model. The utility of candidate $j \in \{A, B\}$ is given by

$$U_j = \begin{cases} 
\lambda + (1 - \lambda)u_j(|\tilde{x}_j - x_j|) & \text{if } j \text{ wins,} \\
(1 - \lambda)u_j(|\tilde{x}_j - x_k|) & \text{if } k \neq j \text{ wins,}
\end{cases}$$

where $\tilde{x}_j$ is $j$'s ideal policy, $u_j$ is a decreasing, concave function, the value of holding office is scaled to 1 with the weight given to holding office, $\lambda$, exogenously given. Candidates are win-motivated when $\lambda = 1$, policy-motivated when $\lambda = 0$ or have mixed motives when $0 < \lambda < 1$. Uncertainty is modelled directly by a symmetric distribution function $F$ (with associated pdf $f$) on the median voter’s ideal point assuming no valence uncertainty (the distribution function $F$ then plays the role of the function $H_\mu(\cdot)$ in Section 3.3). Candidates maximize their expected utility given this uncertainty.

Groseclose first shows that when parties are purely win motivated ($\lambda = 1$), then when $\vartheta = 0$ there is a convergent Nash equilibrium in pure strategies at the median voter’s expected ideal point (a special case of Theorem 7) and when $\vartheta > 0$ there is no such equilibrium. The intuition behind the non-existence result when $\vartheta > 0$ is simple. While candidate $A$ prefers to adopt the same policy as $B$ to capitalize on its valence advantage so as to win with certainty, $B$ has to move away from $A$ to have even a small chance at winning. Hence, no matter what positions the candidates adopt, at least one wants to move. This reasoning extends to multi-dimensional policy spaces ($d > 1$), and to the vote motivation objective and illustrates the knife-edge property of the Downs and Wittman results when valence is introduced.$^{20}$

$^{20}$Aragones and Palfrey (2002) solve this model when candidates choose from a finite number of positions and candidate $A$ has an infinitesimal valence advantage over $B$. In general, there is no pure strategy Nash equilibrium. In their unique symmetric mixed strategy equilibrium, candidates randomize over a fairly small number of positions in a region near the expected median. They show that, as the number of positions becomes fairly large—so that the policy space
Groseclose (2001) then studies the case where candidates are not purely office motivated (so that \( \lambda < 1 \)) and where their ideal policies are symmetrically located around the median voter’s expected ideal policy. He does not prove existence or uniqueness but describes the main features of the equilibrium when it exists. We now present the most interesting characteristics of the equilibrium.

He first shows that, as A’s valence advantage increases from 0 to a small amount, A moves towards the expected median (the “moderating frontrunner” effect) while B moves away from the expected median (the “extremist underdog” effect). In equilibrium, a party trades-off the centripetal incentives (moving closer to the center to increase its probability of winning) and the centrifugal incentives (moving towards its ideal policy to increase its utility when winning). Increasing A’s valence advantage from zero moves the cut-point voter (the one indifferent between both parties) further away from A’s policy and closer to B’s. When voters’ utilities are concave enough, the (absolute value) of the marginal utility of the cut-point voter increases at A’s policy, and decreases at B’s. This reinforces the centripetal force for A, and decreases it for B, resulting in both parties moving towards B’s ideal policy.

However, as A’s advantage increases beyond a certain point, A adopts a more extreme position closer to her ideal point. Actually, when A has an infinite valence advantage, A proposes its ideal policy \( \tilde{x}_A \) and wins for sure. For all levels of A’s valence advantage, A adopts a more moderate policy than B, and as A’s valence advantage approximates a continuous space——, the region over which candidates choose positions converges to that of the expected median voter’s position. Moreover, as A’s advantage converges to zero, the equilibrium probability of winning converges to 1/2. This approach shows that the Downsian results are not so knife-edge since the distribution of strategies in the mixed equilibrium converges to the Downsian pure strategy equilibrium. Note that Aragones and Palfrey (2002) study also the continuous framework: although the continuous game is discontinuous they are able to show existence of an equilibrium in mixed strategies. The existence and characteristics of this mixed equilibrium in a continuous framework are further explored by Aragones and Xefteris (2012). Xefteris (2012) investigates the same model while assuming some uncertainty on the valence advantage.
advantage increases, divergence among candidates’ policies increases.

Groseclose also proves the counter-intuitive result that if $A$ has a large valence advantage then $B$’s policy may be more extreme than its blisspoint $\tilde{x}_B$! Intuitively, when $\vartheta$ is large, the cut-point voter is located at a more extreme position than $B$’s policy (even when $A$ proposes its favored policy $\tilde{x}_A$). In that case, $B$ has an incentive to propose a more extreme position, trading off a first-order gain in the winning probability for a second-order loss in utility when it wins the election.

5.2 Valence in multidimensional policy spaces

We now present multidimensional spatial competition models ($d > 1$) with valence. These valence models derive the conditions under which a pure strategy Nash equilibrium exists even when the necessary conditions for the existence a Condorcet winner do not hold. This is in sharp contrast with the generic non-existence results in multidimensional models without valence described in Proposition 2.

5.2.1 Win Motivation in a Deterministic Model

Ansolabehere and Snyder (2000) consider voters with Euclidean preferences,

$$U_i(\theta_i, j, x_j) = -||\tilde{x}_i - x_j||^2 + \gamma \vartheta_j$$

where $\gamma > 0$ represents the weight given to the valence dimension by all voters, with $\tilde{x}_i$ and $x$ being points in the $d$-dimensional space. Candidates maximize their probability of winning and know the valence and policy preferences of voters. Assume candidate $A$ is valence advantaged: $\vartheta = \vartheta_A - \vartheta_B > 0$. They prove that $(x_A, x_B)$ is an equilibrium if and only if (i) the maximum distance between the ideal point of any voter and any median hyperplane is at most equal to $\sqrt{\gamma \vartheta}$ and
(ii) \( r < \sqrt{\gamma d} \) where \( r \) is the radius of the yolk.\(^{21}\) Furthermore, if \((x_A, x_B)\) is an equilibrium then \( ||x_A - c|| < r + \sqrt{\gamma d} \) where \( c \) is the center of a yolk.

In other words, Ansolabehere and Snyder (2000) show that equilibria in pure strategies exist when \( A \) has a large enough valence advantage over \( B \).\(^{22}\) Candidate \( A \) wins the election in all pure strategies equilibria. While the equilibria place no restrictions on the strategies of the low valence candidate \( B \), \( A \)'s policy position must be near the yolk. Equilibrium existence requires that voters’ ideal points be close enough to any median hyperplane. They conclude that pure strategy Nash equilibria in multidimensional spatial models exist in valence models, and that valence politics and positional politics are inseparable as valence issues are just one aspect of elections that affect candidates’ positions. While advantaged candidates take moderate positions, disadvantaged ones may take moderate or extreme positions.

5.2.2 Vote Motivation in a Stochastic Model

Schofield (2007) builds on Ansolabehere and Snyder (2000) by adding a stochastic partisanship component to their valence model. More precisely, he assumes that voters’ preferences are given by\(^{23}\)

\[
U_i(\theta_i, j, x_j) = -\beta (\bar{x}_i - x_j)^2 + \vartheta_j + \zeta_{ij} \quad \text{for} \quad j \in \{A, B\},
\]

where \( \beta > 0 \) is the weight voters give to the policy dimension with \( \bar{x}_i \) and \( x \) being \( d \)-dimensional points. Voters’ bias for candidate \( j \), \( \vartheta_j + \zeta_{ij} \), is composed of the commonly known valence, \( \vartheta_j \) and of voter \( i \)'s idiosyncratic component \( \zeta_{ij} \) drawn

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\(^{21}\)The yolk is the smallest ball intersecting all median hyperplanes (McKelvey, 1986; Feld et al, 1988).

\(^{22}\)If candidates are vote motivated rather than win motivated, then equilibria in pure strategies typically do not exist unless one candidate has a very large valence advantage.

\(^{23}\)Schofield (2007) studies a setting with \( J \geq 2 \) candidates, but as in the rest of this survey we concentrate on the case where \( J = 2 \).
from a Type I Extreme value distribution with mean zero. When selecting their platforms, candidates observe the valences but not voters’ idiosyncratic biases. As previously, assume that $\vartheta = \vartheta_A - \vartheta_B > 0$ so that candidate $A$ is valence-advantaged. Parties are vote motivated (unlike the win motivation assumed by Ansolabehere and Snyder (2000) in Section 5.2.1).

Schofield focuses on the conditions under which both parties converge to the electoral mean (the mean of voters’ policy preferences dimension by dimension, which is assumed w.l.o.g. to be located at 0)\textsuperscript{24} in a local Nash equilibrium (LNE).\textsuperscript{25} A LNE exists at the electoral mean when the weight voters give to the policy dimensions, $\beta$, is low enough, i.e., when

$$\beta < \beta_0 = \frac{d}{2\sigma^2 \left[ 1 - 2 \left[ 1 - P(0,0) \right] \right]} = \frac{d}{2\sigma^2 \left[ \exp (\vartheta) + 1 \right]} \exp (\vartheta) - 1, \quad (8)$$

where $\sigma^2 \equiv \sum_{s=1}^{d} \text{var}(s)$ measures the “aggregate” dispersion of voters’ ideal points in the policy space ($\text{var}(s)$ being the variance of voters’ ideal points along dimension $s$), and where the probability $1 - P(x_A, x_B)$ that the valence-disadvantaged party $B$ wins, measured at the electoral mean, is

$$1 - P(0,0) = \left[ 1 + \exp (\vartheta) \right]^{-1}, \quad (9)$$

which depends only on the valence difference between the candidates.

Schofield’s result highlights that existence of an equilibrium at the electoral

\textsuperscript{24}Note that the electoral mean is the unweighted mean of voters’ ideal policies. Convergence to the electoral mean also represents convergence the utilitarian maximizing policy (as utilities are Euclidean).

\textsuperscript{25}In Schofield’s setting candidates’ expected vote share functions may fail quasi-concavity (so that none of the usual fixed point arguments can be used to assert existence of a “global” Nash equilibrium in pure strategies). He then uses the concept of a local Nash equilibrium in pure strategies (LNE) defined as a vector of strategies which satisfies the first- and second-order conditions for a local maximum of candidates’ objective function, so that at the LNE no candidate can increase its probability of winning by modifying at the margin its proposed policy. Patty (2005) studies local equilibrium equivalence in a slightly different context.
mean depends on candidates’ valences, on the weight voters give to policies (as opposed to their partisan bias) and on how dispersed voters’ policy preferences are. More precisely, condition (8) becomes easier to fulfil as (i) the relative weight $\beta$ given by voters to policies decreases, (ii) the number of dimensions of the policy space $d$ increases, (iii) voters’s ideal points become less dispersed in the policy space  (i.e., as $\sigma^2$ decreases), and (iv) as the probability of voting for the valence-disadvantaged candidate, $1 - P(0,0)$, increases – which from (9) happens only when the valence difference $\vartheta$ between candidates decreases.\textsuperscript{26}

Even though published earlier, Schofield (2006) extends Schofield’s (2007) multidimensional multi-candidate model by allowing candidates to have both exogenous and endogenous valences. The endogenous valence is generated by the contributions (of time and money) party activists make to candidates to influence their policy positions. Candidates use these resources to present themselves more effectively to voters, thus increasing their endogenous valence. In this extension, voters’ utilities are given by

$$U_i(\theta_i, j, x_j) = -\beta(\bar{x}_i - x_j)^2 + \mu_j(x_j) + \vartheta_j + \zeta_{ij} \quad \text{for} \quad j \in \{A, B\}$$

where $\mu_j(x_j)$ represents the endogenous valence generated by the activist contributions to candidate $j$.

With activists having more extreme policy positions than average voters, candidates must trade-off adopting the more radical policies demanded by activists against the loss of electoral support as they move away from the electoral mean. Schofield (2006) proves that a LNE exists (with candidates positioning themselves where these two forces balance each other), but only when the endogenous valences generated by activists, $\mu_j(x_j)$, are sufficiently concave.

\textsuperscript{26}For empirical applications of Schofield (2007)’s model to various countries under different political regimes and for extensions of his theoretical model, see De Donder and Gallego (2017).
6 Conclusion

This paper surveys part of the recent literature that has moved away from the Downsian approach to electoral competition by exploring other objectives for both parties and voters, by looking at multi-dimensional policy spaces, and by introducing uncertainty about election outcomes at policy platform selection stage. The contributions we survey deliver a rich set of predictions, which are of course very dependent on the assumptions made in each model. We end this survey by summarizing what we consider to be the most important lessons learned from these models.

First, parties’ objectives do matter, but only when parties are uncertain as to the characteristics of voters at the policy platform selection stage.

Second, the way uncertainty is introduced matters a lot for the results, both for the existence of electoral equilibrium and for the characteristics of the equilibrium when it exists (see for example the contrasting results obtained when parties’ uncertainty is about voters’ policy preferences versus partisanship biases).

Third, the literature offers a variety of modelling solutions to overcome, at least in part, the problem of the non existence of a Nash equilibrium in pure strategies in multidimensional policy spaces. Some models (such as those with vote motivation in the stochastic partisanship approach) explicitly state conditions on the data that allow for existence of an equilibrium (see Proposition 5). In other models, such as the PUNEs studied in Section 4.2, an equilibrium exists in many applications, even though there is no general equilibrium existence proof. Equilibria in pure strategies (although of the local variety) also exist in multidimensional models with valence, as seen in Section 5.

Fourth, several models generate the empirically relevant case of non convergence of both parties’ policy proposals at equilibrium. Such a result requires that parties differ from one another. This asymmetry between parties can take the
form of policy motivations (with different preferences over policies) as in Proposition 8, of asymmetric mixed motives (as in Section 4.1 where parties differ in the relative weight they put on winning the elections, as opposed to caring for the implemented policy), of different bargaining weights between intra-party factions (Section 4.2) or of different valences (Section 5).

Fifth, Section 5 shows that valence does matter, but also that the way it is modelled can generate very different results, including quite counter-intuitive ones. For instance, some results from Section 5.1 are, to quote Groseclose (2001), “somewhat unintuitive, since (...) one might expect that valence-advantaged candidates would parlay their advantage into a position that they personally favor more and disadvantaged candidates would do the opposite” while the opposite happens in equilibrium. Interestingly, these counter-intuitive results have strong empirical support.27

Finally, we note that the theoretical literature now offers many models to researchers interested in understanding the electoral equilibrium in specific applied settings. There is clearly no “one-fits-all-purposes” model, and it is up to the researcher to find the one best suited to her or his task.

References


27See De Donder and Gallego (2017) for more on the empirical evidence on this issue.


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