EL SERAFY'S TEMPLATE FOR NONRENEWABLE-RESOURCE ACCOUNTING

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RÉSUMÉ – En 1989, Salah El Serafy a proposé une méthode de comptabilisation d'une ressource non renouvelable sur la base de la valeur actuelle nette. Une réinterprétation et une extension de son analyse permettent ce qu'il appelle une représentation correcte du revenu net à utiliser pour une agrégation précise, « ascendante », depuis des données de projet ou d'entreprise aux comptes sectoriels et nationaux. Un article précédent de Lasserre fournit un soutien intellectuel à la méthode. Ce que l'on peut appeler le modèle d'El Serafy peut être interprété et appliqué à de nombreuses fins, y compris la représentation correcte du revenu et de l'épuisement d'une ressource non renouvelable.

ABSTRACT – In 1989, Salah El Serafy proposed a method of accounting for a nonrenewable resource based on net present value. A reinterpretation and extension of his analysis permit what he calls a proper representation of net income for use in an accurate aggregation, "bottom-up", from project or enterprise data to the sectorial and national accounts. An earlier paper by Lasserre provides intellectual support for the method. What may be called El Serafy's template can be interpreted and applied for many purposes, including the proper calculation of income and depletion of an exhaustible resource.

INTRODUCTION

El Serafy (1989) method of valuation or accounting for a non-renewable resource touches issues at the heart of the theory and practice of economic accounting, including (a) a focus on either of wealth or welfare, (b) the use of dynamic micro- vs. macroeconomic models as a foundation for accounting and (c) the use of shadow prices or market prices as accounting values. Thirty years since the publication of his paper, it is an appropriate time for a re-interpretation.

While El Serafy's method takes an explicit, sector-wide perspective on resource extraction, his approach is better perceived as being at a micro level, based on an analysis of a producing firm. As such, it represents income more accurately than is widely understood. The method applies to the variable profit arising over

time from production using any technology. It can be extended easily to represent the practical decisions that economic accounting seeks to represent as well as a concise summary of their consequences.

Theoretical and empirical support for El Serafy (1989) method is provided by Lasserre (1985) paper on the choice of an irreversible or partially irreversible ("putty-clay") investment in capacity at a mine. Leaving comparatively little opportunity for revision, the capacity choice fixes the future pattern of extraction from the mine. The date of abandonment is endogenous to the choice of capacity and the level of the reserve. While a capacity choice is only implicit in El Serafy's study, his method can be extended.

The extension of El Serafy's methodology is a template for appropriate accounting :

- It treats nonrenewable resources symmetrically to other assets. It thereby stresses the fundamental view of resource economics that resources are forms of capital to be measured like other forms.

- It can be used in the accounting for an optimally or a non-optimally extracted resource. This feature is especially important to resource and environmental economics, in which it is recognized that non-optimality can figure in many ways.

- It can be used for accounting at either market prices or appropriately defined social values.

- It is not limited by modelling assumptions. For example, it can be applied to any extraction technology, including a non-convex technology.

The present paper explains how El Serafy's method can be interpreted. It has a good deal in common with an able precursor by Hartwick and Hagemann (1993) but makes departures from it in the light of Lasserre's model.

1. EL SERAFY'S METHOD

To obtain the broad overview of a typical macro model, an analyst makes strong assumptions, particularly about technology, and thereby sacrifices detail and accuracy. Detail and accuracy, however, are the essence of accounting. By making room for them, El Serafy's method shines. In practice, the sectorial accounts are sums over the realizations at enterprises and projects that comprise the sectors and the sectors are aggregated to form the national accounts. A dynamic, micro perspective is vital. Based on net present value, El Serafy's accounting method is conceptually simple yet can be generalized to encompass any level of realism required.

In the original presentation, beginning at time $t = 0$ a producer exhausts a stock $S(0)$ of a non-renewable resource over $T > 1$ periods. El Serafy assumes

that the resource produces a constant net cash flow or variable profit R in each. The net present value[1] of the resource remaining at time $t \geq 0$, at a constant rate of discount r, is

$$v(S(t)) = \sum_{s=t+1}^{T} \frac{R}{(1+r)^{s-t}}, \quad t = 0, 1, 2, ..., T-1. \tag{1}$$

It is widely understood that, because realizing the present value entails depleting the resource, the entire cash flow cannot be considered to be income. As Hartwick and Hageman stress, though, nor can the entire cash flow be considered to be depreciation. El Serafy's proposal for income in the year t is the value $X(t)$ that if received in perpetuity would have the same present value:

$$\frac{X(t)}{r} = \sum_{s=t+1}^{\infty} \frac{X(t)}{(1+r)^{s-t}} = \sum_{s=t+1}^{T} \frac{R}{(1+r)^{s-t}} = v(S(t)). \tag{2}$$

Equivalently,

$$X(t) = rv(S(t)). \tag{2'}$$

Thus, income is interest on the remaining value of the resource.

For $T < \infty$, $R > X(t)$. Algebraic manipulation yields that

$$R - X(t) = v(S(t)) - v(S(t+1)). \tag{3}$$

Hartwick and Hagemann (1993) also derive equations (2') and (3) and call the latter the *fundamental equation of asset equilibrium*. The difference $R - X(t) > 0$ is the decline in value of the program in period t. It is the *depreciation*, or the *value of the depletion*, of the resource. The sequence $X(t)$, as defined in equation (1) for successive values of t, decreases to zero at T as with a simple mortgage[2].

Indeed, El Serafy's proposal for the definition of income at t, $X(t) = rv(S(t))$, is *Hicksian income number 1* (Hicks, 1946), the value which, if consumed, would leave the present value constant. Hicksian income no. 1 is a definition of income based on wealth accounting and is used widely in economics (e.g. Hill, 2004; Diewert, 2005). El Serafy's method is thus an example of wealth accounting. If in some particular example, it happens that (or is optimal that) cash flow is equal to Hicksian income no. 1 in the period, the present value $v(\cdot)$ does not change in that period.

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1. Herein the convention is adopted that cash flows are received at the end of a time period rather than the beginning. The convention makes some expressions more compact than in the original formulation. There is no loss of generality.

2. See El Serafy (1989, pp. 14-15), diagram and table.

As in El Serafy's model, however, stationarity of value is not necessarily the object or the outcome of the decisions. Hicksian income is a technical measure defined at t for the presumed sequence of net cash flows (variable profits) from the mine. There is no implication that Hicksian income, or some lesser amount, must or should be consumed[3].

Wealth accounting can be distinguished from *welfare* accounting, which measures changes in welfare over time in a macroeconomic model and so takes an important perspective on economic activity. In welfare accounting, the prices are assumed to be the shadow prices for the given social-welfare functional, often a measure of utility discounted at a constant rate of pure time preference.

Conditions of production, however, and not the differences between wealth accounting and welfare accounting, are the focus of the present discussion. An accurate treatment of the conditions of production is essential to both.

2. WEALTH ACCOUNTING

The relevance of wealth accounting stems from Irving Fisher's separation theorem (cf. Hirshleifer, 1958; Becker, 2008), according to which economic actors maximize wealth in order to purchase goods that maximize their dynamic welfare. Wealth accounting can be used to evaluate economic decisions from the point of view of the decision maker. Hamilton and Ruta (2009) observe that "[i]f income equals consumption plus the change in total wealth, then we have a neatly contained and intuitive theory of income and wealth – income equals the return on total wealth..." Their observation is represented above as equation (2').

In their section on "practical wealth accounting", Hamilton and Ruta (2009) derive a shadow price of the resource and use it as the basis of accounting. Let there be a marginal increase in the initial stock $S(0)$. To Hamilton and Ruta, T is fixed and the increase in stock is allocated to increase R equally over periods $t = 1, 2, ..., T$, giving rise to a particular shadow price. In this case, it must be possible to increase output at the margin in each period.

Wei (2015) differs with Hamilton and Ruta in the calculation of the shadow price. Since net cash flow R is represented as being constant in El Serafy's paper, Wei reasons that it is not possible to increase cash flows over $1, ..., T$. One might say that, if price is constant, then the level of output is limited somehow, such as

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3. The point is crucial in the context of resource accounting. El Serafy's paper is in a volume devoted to accounting for "sustainable development". His model suggests that the mine owner can *sustain* an income of $X(t)$ from t onwards. The sustainability result, however, cannot be generalized to an entire economy because changing the path into a sustained path entails changes in prices, assumed here to be given. For an economy, a measure akin to $X(t)$ has been called the *stationary-equivalent income* (Weitzman, 1976) and is distinct from sustainable income. Sustainable income is consistently defined in a maximin program (Solow, 1974; Hartwick, 1977; Fleurbaey, 2015). Only if maximin prices are decentralized is $X(t)$ truly the sustainable income. (See Cairns and Martinet, 2014). El Serafy's accounting method applies for any prices, be they market prices, optimally decentralized prices for a welfare objective, or the prices of a maximin objective.

by sunk capacity as discussed by Lasserre. Therefore, the marginal unit cannot be extracted until after the final date T. In his evaluation, Wei obtains a different expression for the shadow price.

Hamilton and Ruta move from the particular case of an optimal economy to more general cases, viewing El Serafy's analysis as applying solely to a non-optimal path. Since a main purpose of economic accounting is to evaluate non-optimal paths, economic accounting should apply to them. A general method should, then, eschew the use of optimality conditions because they apply to a special case. Lasserre's insights are useful in interpreting the findings of Hamilton and Ruta and of Wei: One of the questions to be dealt with in examining El Serafy's template in the context of production conditions is the role of shadow prices, which are marginal values that may or may not be related to optimality conditions (Dasgupta and Mäler, 2000).

3. A KEY EXTENSION

The papers by Wei and by Hamilton and Ruta take for granted that a distinctive feature of El Serafy's method, rather than an expositional convenience, is that the net cash flows remain constant through to exhaustion at a finite time. In the context of the economics of exhaustible resources going back to Gray (1914) and Hotelling (1931), the assumption of constant net cash flows may seem anomalous. On the contrary, we argue herein that (a) the assumption is not unnatural but that (b) in any case it is not an essential element of El Serafy's accounting method.

The imposition of the constancy of the cash flows suggests that there is no decision about extraction at any time $t > 0$. In effect, the resource is irreversibly committed, or sunk, at time 0 and the declining stock remains sunk until time T. What makes it sunk? How do the net cash flows come to be decided?

In reality, which El Serafy seeks to represent, mining is among the more capital-intensive industries. A highly significant generalization of the production process is that the reserve, $S(0)$, is developed through the application of non-resource capital at $t = 0$. Development commits the reserve as well as the other forms of capital to future production in a particular way (Cairns, 2001, 2009). All of the forms of capital, including the reserve, are transformed and subsumed into a new, composite asset called a mine. The composite (as opposed to the various component assets in isolation) is what produces the flows of output and hence the net cash flows.

For illustration, we make some stark, but not misleading, assumptions about the operation of the mine. None of these assumptions is necessary for the application of the template[4].

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4. The relaxation of assumptions makes the analysis of net cash flows more complicated but conceptually the same. If, for example, new capital (including new parts of the resource) can be added it is accounted incrementally. Lasserre (1985) stresses investment when a mine is newly opened, but also allows for expanding the capacity at a later date. For an application to oil, see Smith (2014)

- The entire reserve is developed at time 0. No new capital is added during the life of the mine.

- The capital and reserve are sunk for the life of the mine and afterward have no alternative use.

- There is one type of produced capital.

- The (productivity of the) produced capital does not decay or deteriorate.

The value of the resource at $t = 0$, $\Psi(S(0))$, is found by subtracting the value of the produced capital $\Phi(I)$ from the net present value $V(S(0), I)$:

$$\Psi(S(0)) \triangleq \sum_{t=1}^{T} \frac{R_t}{(1+r)^t} - \Phi(I) = V(S(0), I) - \Phi(I). \tag{4}$$

If the sunk capital I is assumed to provide an upper bound to extraction from the mine (Lasserre's capacity), then under the above assumptions extraction is constant (Cairns, 2001). If price and average cost are also constant (e.g., if the planner holds the price of gold at \$35/oz.) the net cash flow is constant. A natural special case, then, justifies the pattern in El Serafy's example.

In the context of Lasserre's model of putty-clay capital, Hamilton and Ruta's model allows for a change in the capacity before the capital is committed and thus points to the long run. (It does not envisage a change in the endogenous date of exhaustion, however.) Wei's model does not admit a change in capacity and points to the short run.

Equation (4) is a long-run valuation of the *resource*, before the decision to sink the capital. El Serafy's analysis, like Wei's, can be viewed as commencing just after the investment in capacity. The calculation in equation (1) finds a short-run value of the *mine*, $V(S(0), I)$, not of the resource, $\Psi(S(0))$. Once the investment is sunk, the omelette of a mine cannot be unscrambled; for $t > 0$ there exists no value $\Psi(S(t))$ that applies to the resource alone.

When costs are sunk, the assignment of a user-cost schedule and an implied depreciation schedule for the mine, as sought by El Serafy and by Wei, is what Diewert (2005) calls "the fundamental problem of accounting". There are an uncountably infinite number of consistent user-cost schedules for the resource that complement the schedules for other capital in net cash flow (Cairns, 2004, 2009, 2013). In general, the user cost of the resource is not related to its shadow value, $\partial V(S(t), I)/\partial S(t)$. Nor is it related to the shadow values used in dynamic macro, sectorial or welfare models of national accounting that use Hotelling's rule, which does not hold in a model with capacity constraints (Cairns, 2001; Davis and Cairns, 2007).

An appropriate schedule of user costs for the resource, from which depreciation and income can be calculated, allows for the recovery of the initial values

$\Phi(I)$ and $\Psi(S(0))$ from the net cash flows over the mine's life. The minimum conditions required to achieve this recovery of cost, which any valid schedule satisfies, are the following:

1. The discounted value of user costs attributed to the resource in the periods $t = 1, ..., T$ is equal to $\Psi(S(0))$, and to the produced capital is $\Phi(I)$.

2. User costs in any period are nonnegative.

3. Since the composite of resource and capital are the items of capital that produce the net cash flows, the sum of the user costs attributed to the resource and to produced capital in any period is the net cash flow R.

Conditions 1 and 3 impose two types of "adding up" that are pivotal to accounting consistency.

4. MARGINAL CONDITIONS AND SHADOW PRICES

Even if the resource is optimally exploited (and even in some notional "nth best"), conditions 1 to 3 do not rely on optimality conditions or shadow values. If, as in practice, the sectorial and national accounts are obtained by adding up results from individual enterprises, then economic-accounting magnitudes – non-unique but limited by the three conditions – do not utilize shadow values. Capitalized values exist at time 0, before they are combined into a mine, namely, $\Phi(I)$ for the non-resource capital and $\Psi(S(0))$ for the reserve. If there are non-linearities[5], the total value at any time is not equal to the assets valued at their marginal (shadow) values (Cairns, 2001). Even though there exist marginal *shadow prices* for each there is not a unique *accounting price* for either – as befits sunk and hence untraded assets (for $t > 0$ in the model). According to microeconomic theory, short-run extraction cost is a function of the quantities of the two assets, $c(q(t), S(t), I)$, and is not referred to any input price. There is a unique value of the *mine* at any $t = 0, 1, 2, ..., T-1$, namely, the remaining net present value, $V(S(t), I)$.

Conditions 1, 2 and 3 imply that equations (1), (2) and (3) provide a template for accounting for $\Psi(S(0))$, $\Phi(I)$ and $V(S(0), I)$ that is analogous to that for $v(S(0))$. The title of El Serafy's article is well chosen: an extension of his model is the "proper" dynamic model for evaluating nonrenewable-resource use. It applies directly to the evaluation of the mine throughout its T periods of production. User costs *assigned* to capital and the resource, and hence values of depreciation and income obtained by manipulating their present values as in equations (2) and (3), also take the pattern of the template and are extensions of it. The presentation is a model, a simplification of reality, but one that is devised to correspond to the decisions that are taken in this industry. The method can be applied to any

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5. Contrary to the traditional assumption of weak or strong convexity, in practice $\Phi(I)$ is typically strictly concave.

number of capital goods and, by accounting for incremental values, to further investment or disinvestment during the productive life of the reserve. For example, Cairns (2004) presents an optimization model in which two produced assets act in conjunction with a non-renewable resource; accounting is according to the above extension of El Serafy's method.

Lasserre (1985) assumed that the mine was optimally developed. El Serafy's method applies to a mine operating under such an assumption. It also applies to a non-optimally run mine, as is essential if accounting is to be useful. It bears stress, then, that in El Serafy's method there is no use of marginal or shadow values. The fact is not related to the constancy of net cash flows in equation (1). El Serafy assumes that R is constant solely for illustration (see p. 16, 17). To perform the algebraic manipulation of discounted net cash flow to obtain depreciation and income, it is not necessary that any variable remain constant. If present value at time t is written

$$v(S(t)) = \sum_{t+1}^{T} \frac{R_s}{\Pi_{\tau=t}^{s}(1+r_\tau)} \tag{5}$$

then whatever the pattern of cash flows R_s, whatever the value of T (including infinity) and whatever the pattern of interest rates r_t may be, income at t is given by $r_t v(S(t))$ and depreciation (of the mine) by $v(S(t)) - v(S(t+1))$. The latter expression is Hotelling (1925) and Samuelson (1937) definition of depreciation.

The only input to the accounting method is observable net cash flows. That these net cash flows and the interest rate can vary through time and that the horizon need not be considered to be finite make the template, an extension of equations (1) through (3), applicable to the accounting of any enterprise, not just a nonrenewable resource[6].

Renewable solar or wind energy, for example, involves

- variable, intermittent outputs of energy at varying prices, with varying net cash flows that can be aggregated to hourly, monthly, annual or other appropriate, discrete time periods to be used as the terms in equation (5) as well as;

- irreversible investments in solar panels or windmills, which can be evaluated as in equation (4).

Pricing and production of these outputs are often criticized as being non-optimal. When they are, a method of evaluation still is (or especially is) required. If there are replacements, as for replanting a plantation forest that provides amenities, then an economic accountant must be careful to make the algebra represent the decisions of the enterprise.

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6. Accounting numbers for any type of enterprise are collected at discrete intervals.

5. SMOOTHING

Accounting is a method of organizing data for input to valuation or to further study. Herein, the basis is El Serafy's method, for which the lumpiness and non-convexity inherent in investing and producing from a mine are explicitly evaluated. Furthermore, in reality mineralization occurs in many dispersed deposits. Each is exploited using its own, specific investment of sunk capital as well as variable inputs. The non-convexity is not smoothed but is made more prevalent when multiple deposits of substantial size enter production at different times. However, an inappropriate use of the analysis of a steady state, with a new entry taking the place of firms that are leaving because of exhaustion of their reserves, can recommend a method of accounting that is a basis for a spurious smoothing of the data. It is significant that adherence to El Serafy's template for accounting and then forming the sectorial accounts correctly by summation obviate this spurious smoothing.

By equation (4), the value produced by a developed mine i is the discounted net cash flow of producing q_{it} in period t, $R_{it}(q_{it})$, net of the investment cost $\Phi(I_i)$:

$$\Psi(S_i(0)) = \sum_{t=1}^{T} \frac{R_{it}(q_{it})}{(1+r)^t} - \Phi(I_i). \tag{6}$$

This equation holds for a choice of I_i (optimal or not), assumed to remain fixed for the life of the mine.

In a steady state, for each mine i at each date t, output q_{it} is a constant, q; levels of investment and revenue and all other variables are also constant. (This form of assumed constancy is because of stationarity in the steady state and is not the constancy assumed for purely expositional purposes by El Serafy; as will be shown below, the former contributes to spurious smoothing of sectorial accounts.) In particular, for the given choice criterion of the firm, there is a function ψ for which $I_i = \psi(q)$. For each representative firm, then,

$$\Psi(S) = \sum_{t=1}^{T} \frac{R(q)}{(1+r)^t} - \Phi(\psi(q)). \tag{7}$$

Entering at time t are n new firms, each incurring a capital cost $\psi(q)$. Their capacity adds to that of $n(T-1)$ firms that are continuing to exploit developed reserves. Total industry cash flow at t is

$$n(T-1)[pq-c(q)]+n[pq-c(q)]-n\psi(q) = nT\left[pq-c(q)-\frac{\psi(q)}{T}\right]. \tag{8}$$

In this steady state, with n and T constant, it can seem from equation (8) that accounting for each of the nT firms, which entered from dates $t-(T-1)$ up to t, can be done as if cost were all variable cost, $c(q)+\psi(q)/T$, which is differentiable

if the functions c and ψ are. Moreover, let $Q = nTq$ represent industry output. Then, for obvious definitions of C and K, total industry cost at t can be put in the form

$$nT\left[c(q) + \frac{\psi(q)}{T}\right] = \left[nTc\left(\frac{nTq}{nT}\right) + nT\frac{\psi\left(\frac{nTq}{nT}\right)}{T}\right] \triangleq C(Q) + K(Q). \qquad (9)$$

This (spurious) representation of cost can be taken as a variable and differentiable function of industry output Q.

At each time t in the steady state, the spurious industry costs (and hence cash flows as well), represented as variable, are equal to the costs (and cash flows) in the true representation of the technology, in which capital costs are sunk and specific to the n entering firms.

The assumption of the steady state is strong. However, in a mature mineral region with a large number of small firms or a small number of large firms with many prospects, growth or decline is fairly steady and there are continual additions to the producing projects and occasional retirements. Even if investment and output levels vary among firms, in the accounting for the industry aggregate, costs can appear to be smooth, especially if results for the various mines of large firms are consolidated in their enterprise accounts. Accounting for the industry might then be based on disaggregating equation (9) to equation (8) proportionally to output rather than aggregating equation (6) and its lumpiness to the level of the sector.

By assuming convexity and smoothness in order to be tractable, dynamic macro and welfare models can thus abstract from a pivotal feature of resource extraction. In so doing, they can short-circuit the investment decisions and their costs, which economic accounting is intended to evaluate.

In addition, accurate accounting for extractive industry must take geological features into account. For a conventional oil-and-gas reserve, in which El Serafy was primarily interested, or a shale reserve, the assumption of constant net cash flows R does not hold. Given an initial investment I in development of a reserve, extraction is constrained by natural decline of the pressure of the oil and gas remaining in the reserve. Once the investment I is chosen, the flow of output is in essence predetermined and not chosen (Adelman, 1990; Smith, 2014). Still, El Serafy's template applies to accounting for the value of the composite of reserve and invested capital, based on the net present value of the anticipated cash flows. Furthermore, it can easily be generalized to determine (non-unique) user costs of the resource and of the other capital using conditions 1, 2 and 3 (Cairns, 2009).

The method of aggregation is key to understanding the economics of this industry, its equilibrium and its accounting. As in practical national accounting, accounts at the mine level are summed, bottom-up, to the sectorial and finally the

national levels. The productive conditions discussed herein also apply for welfare accounting. On the basis of a thought experiment, one might reason that if a welfare model were to represent technology and geology accurately, accounting for each mine would take the same form as in the template, using prices established in the model's resource-allocation mechanism (cf. Dasgupta and Mäler, 2000). However, production is disaggregated, non-convex, and lumpy; it is intractable in a general equilibrium, welfare model.

In the next step of analysis, both welfare and wealth accountants must come to grips with these conditions. Therefore, they must also deal with the inapplicability of marginal shadow prices. There is, for example, no price p by which the value of a nonrenewable-resource stock S can be written pS.

CONCLUSION

El Serafy's template remains within the neoclassical tradition and is consistent with contemporary advances in the study of irreversible investment (under certainty). It is built from elementary, primitive notions and has a stark simplicity. It can confront at the micro level mathematical properties of technology and geology that are hard to represent in macro models. Its results can be summed to contribute to the sectorial and national accounts.

The accounting must be as it is in the template because the economics of the industry is as it is. The following are properties of the template for economic accounting for nonrenewable resources:

- As has been observed by many, it is incorrect to treat all net cash flows from the resource as income.

- It is also incorrect to treat all net cash flows from the resource as depreciation, the value of depletion.

- Resources are complementary with other sunk assets in extraction and in accounting. They form a new, composite asset that subsumes its components.

- Accounting is flexible in accommodating any number of assets and additions through time.

- Accounting is disaggregated to the level of decisions, to firms and mines, and summed to the sectorial level.

- Decisions are lumpy.

- Marginal (shadow) values are not used.

- There are an uncountably infinite number of valid depreciation or user-cost schedules for the component assets.

- The template is valid for accounting for both non-optimal programs and the special case of optimal programs.

- The link to welfare is indirect, through Fisher's separation theorem.

El Serafy's paper reminds us that the national accounts are not an independently standing macroeconomic system but are aggregates of sectorial and ultimately of microeconomic accounts. El Serafy's method is a template for the dynamic analysis of accounting for extractive industry.

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