Economic Policy when Models Disagree*

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This paper proposes a general way to conceive public policy when there is no consensual account of the situation of interest. The approach does not necessitate a representative policymaker's utility function (as in the ambiguity aversion literature), a reference model (as in robust control theory) or some prior probability distribution over the set of supplied scenarios (as in Bayesian model-averaging). It requires instead that the value of a remedy's projected outcomes agrees with the willingness-to-pay to escape the current situation. Policies constructed in this manner are shown to be effective, robust, simple and precautionary in a precise and intuitive sense.

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We picture facts to ourselves.
A picture is a model of reality.
- Ludwig Wittgenstein (1922) -

I. Introduction

Models are an ever-present input of decision and policy making. Be they very sophisticated or not, they always are partial representations of reality. The same object might therefore admit different models. Well-known current examples include global warming and its various impact assessment models, such as the DICE model of Nordhaus (1994, 2007) and the PAGE model used by Stern (2007, 2008), and macroeconomic policy, with its competing DSGE models that respectively build on the New Keynesian framework (see, e.g., Clarida et al. 1999; Woodford 2003) or the Real Business Cycle view (see, e.g., Cooley 1995).[1] Due to theoretical gaps, lack of data, measurement problems, undetermined empirical specifications, and the normal prudence of modelers, such episodes of model uncertainty might often last beyond any useful horizon.[2] Meanwhile, policymakers

[1] The "Dynamic Integrated model of Climate and the Economy" (DICE) is a global-economy model that explicitly considers the dynamic relationships between economic activity, greenhouse-gas emissions and climate change. The "Policy Analysis for the Greenhouse Effect" (PAGE), developed by Hope (2006), generates emission-reduction costs scenarios for four world regions, acknowledging that some key physical and economic parameters can be stochastic. There are many other models addressing the economics of global warming (see, e.g., Loulou et al. 2010, Manne et al. 1995; Nordhaus and Yang 1996; Nordhaus and Boyer 2000; and Stern 2007, chapter 6). Disagreements between modellers have to do with microfoundations and descriptive accuracy (the so-called "top down" versus "bottom up" models), discounting, technological innovation, and the treatment of risk and uncertainty (see, e.g., Heal 2008). Dynamic Stochastic General Equilibrium (DSGE) models, on their part, differ mainly in their microfoundations and the way they capture price and wage adjustments.

[2] As Watson (2008, p. 37) pointed out, for instance: "In the foreseeable future (next 20 years) climate modelling research will probably not materially decrease the uncertainty on predictions for the climate of 2100. The uncertainty will only start to decrease as we actually *observe* what happens to the climate." [Emphasis added] In his recent appraisal of climate-change policy, Helm (2008, p. 236) makes a similar point: "Science, too, takes time: as noted at the outset, we are condemned to uncertainty over the relevant time period within which action needs to be taken."

will be expected to act based on analyses, scenarios and forecasts which can be at variance from each other.

Economists have recently devoted significant efforts to assist policy making in such circumstances.[3] At least four approaches can be found in the literature at the moment:[4] model averaging, seeking undominated policies, ambiguity aversion, and robust control. The first one draws usually on Bayesian decision theory, thanks in part to new means for constructing prior probability distributions (Raftery et al. 1997; Chamberlain 2000; Fernandez et al. 2001; Billot et al. 2005), and has been advocated by a number of macroeconomists (see Sims 2002, Brock et al. 2003, and the references therein). Recent extensions that build on seminal works by Gilboa (1987) and Schmeidler (1989) now admit non-additive weights, which is sometimes hard to avoid in dealing with deep uncertainty (notably in risk assessment and portfolio allocation, as argued by Bassett et al. 2004). The second route, taken for instance by Manski (2000) for the selection of treatment rules, dispenses with prior distributions, seeking only policies that cannot be outdone in at least one model. The third way acknowledges instead that several prior distributions might be plausible at the same time; it then develops decision criteria - such as Gilboa and Schmeidler (1989)'s static maximin criterion or the recursive utility models of Epstein and Schneider (2003), Maccheroni et al. (2006), and Klibanoff et al. (2009) - that fit what

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[3] The first recognition of the importance of model uncertainty for the evaluation of macroeconomic policy actually dates back to Brainard (1967).

[4] One might also add *exploratory modeling* to this list. Pioneered by Bankes (1993), exploratory modeling combines human judgment with systematic interactive computer experiments on a given family of plausible models in order to shed light on policy choices. This approach is currently used in long term policy analysis (see Lempert et al. 2003, for instance). It relies heavily on information technology, but still lacks some economic foundations.

are considered to be reasonable patterns of individual behavior in this case (as they have been documented since Ellsberg 1961's seminal article). The maximin approach has found many applications, notably in finance (see, e.g., Cont 2006, Garlappi et al. 2007, and the references therein). Robust control, finally, builds on engineering (optimal control) tools for finding policies that will put up with any perturbation of a given reference model.[5] It was persuasively introduced in macroeconomics by Hansen and Sargent (2001, 2008); some applications exist as well in natural resources economics (Roseta-Palma and Xepapadeas 2004; Vardas and Xepapapeas 2009).

These methods, however, are subject to important caveats. As Levin and Williams (2003) argued in macroeconomics, and Al-Najar (2009) recently showed in a general setting, there might often be no single reference model since key issues (such as expectations formation, inflation persistence, fiscal multipliers, or the regional impacts of global mean temperature changes) may never be settled by theoretical investigation or empirical evidence; this reduces the scope of robust control techniques.[6] The main alternatives, i.e. Bayesian model-averaging and multiple-prior decision making, on the other hand, call for probabilistic beliefs over a collection of models or scenarios, which might also prove to be far-fetched in many situations. A major contribution of the recent literature on belief formation has actually been to pin down conditions in which entertaining probabilistic

[5] In physics, a "perturbation" means a secondary influence on a system that causes it to deviate slightly. Hansen and Sargent (2008) define the word "slightly" as lying within a certain range of the reference model, where distance is measured by an entropy-based metric.

[6] To be accurate, robust control does allow policy makers to hold divergent beliefs concerning a model's misspecifications, but it requires that everybody consider the same benchmark model (see Hansen and Sargent 2008, part IV).

beliefs is hardly achievable or even rational (see the recent survey by Gilboa et al. 2008).[7] Besides, the available criteria based on ambiguity aversion can be questioned on several grounds. The maximin criterion (which is also used in robust control) really corresponds to an extreme form of uncertainty aversion (Adam 2004); the more sophisticated ones are not yet operational (especially for eliciting and capturing *collective* preferences); the normative value of ambiguity-averse preferences or nonexpected utility remains debatable (Al-Najar and Weinstein 2009; Wakker 1988); the association made between ambiguity aversion and concerns for robust policies seems unwarranted (Nehring 2009); and criteria which explicitly seek to convey individual rationality may demand too much and lead to inferior outcomes in what is essentially a political process involving many heterogenous agents.[8] Falling back at last on undominated policies, in order to do away with aggregation while acknowledging several distinct models, may be quite disappointing, for such policies can be numerous and are allowed to do poorly under some scenarios.

Our goal in this paper is to set out a new approach which avoids these shortcomings. The proposed scheme, which is sketched in Figure 1 and will be formalized in section III, borrows several core elements from Tinbergen (1952)'s theory of economic policy.[9] In this

[7] Aragonès et al. (2005) show that complexity, for example, can be one reason for this. A group of experts might also fail to hold a common prior if the set of models or scenarios is sufficiently large (see Cripps et al. 2008).

[8] Related to the latter point is the recent analysis by Osborne and Turner (2010) who show that, if citizens hold similar preferences but information about the relevant issue is widely dispersed, a referendum leads to a better policy decision than a cost-benefit analysis does; for "(...) a cost benefit analysis elicits cardinal information about preferences whereas a referendum elicits only ordinal information." (p. 178)

[9] For an historical perspective, literature review and appraisal, the reader may consult the articles by Hallett (1989), van Velthoven (1990), Eggertsson (1997), and Acocella and Di Bartolomeo (2007).

setting, a model brings together endogenous and exogenous variables, and some policy instruments (the short-term interest rate, say, or a carbon tax). Let different models involving the same policy instruments be simultaneously relevant to policymakers. For initial values of those instruments and the exogenous variables, each model $i = 1, ..., n$ delivers a (possibly dynamic and stochastic) scenario or forecast ω_i.[10] In this context, a *policy rule* Φ is a prescription on the utilization of the policy instruments that prompts a revision of all scenarios. The challenge is to design a suitable rule.

Insert Figure 1 about here.

Suppose that each original scenario ω_i is given a score u_i via a mapping U, and that the revised scenarios $\omega_1', ..., \omega_n'$ must go through an overall policy assessment $\mathrm{v}(\omega_1', ..., \omega_n')$ expressed in monetary units. Call a policy rule *effective* if its outcome receives a positive assessment whenever the score of at least one initial scenario fell short of some pre-established objective. We show in Section IV that an effective policy rule exists if and only if a shadow price $\pi(u_1, ..., u_n)$ can be put on each configuration of scores so that

$$\mathrm{v} \circ \Phi = \pi \circ U \quad . \tag{1}$$

This comes directly from a generalization of Farkas's Lemma - a statement central to linear programming and convex optimization - due to Craven (1972). Once an appropriate shadow price schedule π is available, a policy Φ can then be searched for by solving the above equation (sufficient conditions for this to succeed are supplied in Section IV).

The scores u_i, assessment v and shadow price π should be regarded as regular fea-

[10] For simplicity, we assume all along that there is a finite number of models. Our framework does allow the set of models to be (countably or uncountably) infinite, but at a technical cost.

tures of a policy process. Scores are indeed inherent in rule-based policies such as the Taylor Rule (Taylor 1993) or the Kyoto Protocol, where they convey positive or negative deviations from some intended GDP level and inflation rate or some emission reduction target respectively. If a scenario is a probability distribution over outcomes, its respective score could also be an expected utility, a certainty equivalent, or some measure of riskiness that depends only on the distribution itself (such as the "objective" risk measures developed by Foster and Hart 2009, or Artzner et al. 1999). As to the mappings π and v, they may correspond respectively to an ex ante social cost function and an ex post social welfare function, the former focusing on prospects and the latter on the consequences of policies. Such items are now commonly used in environmental policy assessments, for instance (Freeman 1979). They can be based on expected or nonexpected utility, or on some ambiguity-averse criterion. They might also come down to voting rules, or to some quantitative account of the opinion shared by members of an official board (perhaps after several discussion rounds, as reported by Sims 2002, and Leeper and Sargent 2003).

It seems natural to interpret the shadow price π as expressing the policymakers' joint willingness-to-pay for avoiding exposure to scenarios with marks in the range $\{u_1, ..., u_n\}$. Equation (1) then says that *an effective policy must make the value of its projected outcomes agree with the willingness-to-pay to escape the current situation.*

To fix ideas further on this approach, the next section gives an example of what it does in comparison to previous methods. The formal framework and general construction of policy rules are then laid out in Sections III and IV respectively. Major normative properties of these rules - such as robustness, and simpleness - are shown and discussed in Section V; let us stress that these properties are not postulated a priori but are instead *derived* from the construction. Section VI illustrates the approach further by sketching two key potential applications (in banking regulation and climate policy respectively). Section VII concludes with final remarks regarding some possible refinements of the proposed scheme.

II. An Example

Suppose there are two accepted models of an economy, none of which can be taken as a benchmark. Each model $i = 1, 2$ generates forecasts of aggregate wealth which take the form of normal distributions $N(a_i - z; (1-z)\sigma_i^2)$ with mean $a_i - z$ and variance $(1-z)\sigma_i^2$. The parameters a_i and σ_i^2 are exogenous and specific to each model. The variable z, which is scaled so as to belong to the interval $[0, 1]$, refers to variance-reducing policies (such as the number and levels of some automatic stabilizers) that cost one unit of expected wealth per unit of decrease in volatility. Let $a_1 > a_2$ and $\sigma_1^2 > \sigma_2^2$, so the first model reckons a larger average wealth but also greater volatility for any given policy z.

Assume the policymakers' collective preferences over aggregate wealth are representable using the constant-absolute-risk-aversion (CARA) utility function $u(x) = -e^{-\alpha x}$ with coefficient of absolute risk aversion α. It is well-known that ranking the forecasts of models $i = 1, 2$ based on the expected values of a CARA utility function amounts to comparing the certainty equivalents

$$CE_i(z) = a_i - z - \alpha\frac{(1-z)\sigma_i^2}{2} = (a_i - \alpha\frac{\sigma_i^2}{2}) + z(\alpha\frac{\sigma_i^2}{2} - 1) \ , \quad i = 1, 2.$$

Undominated policies will then generally take the form $z = 1$ (if $\alpha\frac{\sigma_i^2}{2} > 1$ for some i) or $z = 0$ (if $\alpha\frac{\sigma_i^2}{2} < 1$ for some i). Alternatively, Bayesian policymakers who hold that model 1 is right with prior probability p will choose z to maximize

$$pCE_1(z) + (1-p)CE_2(z) = z\left[p\alpha\frac{\sigma_1^2}{2} + (1-p)\alpha\frac{\sigma_2^2}{2} - 1\right] + \ \textit{a constant}$$

and be thereby led to select $z = 0$ or 1. When $\alpha\frac{\sigma_1^2}{2} > a_1 > 1$ and $\alpha\frac{\sigma_2^2}{2} < 1 < a_2$, however, such dichotomous policies will perform rather poorly under one model.[11]

In the latter case, by contrast, the maximin policy z^* sits at the intersection of the curves $CE_1(z)$ and $CE_2(z)$, for any set of priors that includes $p \approx 1$ and $p \approx 0$. This action certainly limits the policy maker's exposure to regrettable outcomes if either scenario turns out to be the wrong one. But it may seem overly cautious to several people, especially if one model prefigures a very large return from modifying z^* slightly.

Turning now to this paper's approach, consider for simplicity the situation depicted in Figure 2, where $CE_1(z^*) > -a_1 + \alpha\frac{\sigma_1^2}{2}$.

Insert Figure 2 about here.

Suppose $z = 0$ is the current policy, so the initial forecasts are in fact $\omega_i = N(a_i; \sigma_i^2)$. Ascribe the welfare scores $u_i = a_i - \alpha\frac{\sigma_i^2}{2}$ to these forecasts; let the revised scenarios be $\omega_i' = N(a_i - z; (1-z)\sigma_i^2)$; and take

$$\mathrm{v}(\omega_1', \omega_2') = \min[a_1 - z - \alpha\frac{(1-z)\sigma_1^2}{2}\ , a_2 - z - \alpha\frac{(1-z)\sigma_2^2}{2}]$$

as the *ex post* policy assessments. If the function

$$\pi(u_1, u_2) = -\min[u_1, u_2],$$

[11] Obviously, the recommended actions took values 0 or 1 because we assumed the cost of policy was linear. Supposing instead a convex cost $c(z)$ could have resulted in remedies $0 < z < 1$, but the contrasts we want to emphasize with these standard approaches to model uncertainty would then fade away.

captures the policymakers' combined willingness-to-pay to avoid the existing welfare possibilities $\{u_1, u_2\}$, then solving equation (1) amounts to seeking a policy $z^\bullet$ such that

$$\min[a_1 - z^\bullet - \alpha\frac{(1-z^\bullet)\sigma_1^2}{2}\ ,\ a_2 - z^\bullet - \alpha\frac{(1-z^\bullet)\sigma_2^2}{2}] = -\min[a_1 - \alpha\frac{\sigma_1^2}{2}\ ,\ a_2 - \alpha\frac{\sigma_2^2}{2}]\ .$$

This yields two candidates $z_A^\bullet$ and $z_B^\bullet$. These policies will not do as well as z^* in the worst case, of course. But their respective return will never be inferior to the policymakers' subjective quote $\pi(u_1, u_2)$ to escape the current uncertain situation. In the above figure, moreover, $z_B^\bullet$ may produce a much higher certainty equivalent than z^* if model 1 turns out to be right.[12]

Policies like $z_A^\bullet$ and $z_B^\bullet$ could have been generated as well through the maximin approach, using a restricted set of priors (based on the axioms in Gajdos et al. 2004, for instance) that excludes $p = 0$ and $p = 1$, or invoking one of the recent criteria for decision making under ambiguity. Our method, however, does not involve a selection of prior distributions (which could require an infinite regress in beliefs) or an exact encoding of ambiguity aversion. The willingness-to-pay $\pi(u_1, u_2)$ and policy evaluations $\mathrm{v}(\omega_1', \omega_2')$ should be viewed as directly observable components of the policy process that do not need to be traced back to a particular utility function.[13]

[12]If the curve $CE_2(z)$ were almost horizontal and crossed the actual $CE_1(z)$ and $\pi(u_1, u_2)$ from above at points $z_A^\bullet + \varepsilon$ and $1 - \delta$ (ε being a tiny positive number), then the maximin approach would prescribe action $z^* = z_A^\bullet + \varepsilon$ while our procedure would rather recommend $z_A^\bullet$ and $z_B^\bullet = 1 - \delta$. We submit that most policymakers would ultimately choose the latter.

[13]Instead of the above "Leontief" social preferences, the mappings v and π could express the outcomes of approval and disapproval voting respectively. In this case, one may have $\pi(u_1, u_2) = -1$ if u_1, $u_2 \geq 0$, and $\pi(u_1, u_2) = 1$ otherwise; $\mathrm{v}(\omega_1', \omega_2') = 1$ if the corresponding certainty-equivalent u_1', $u_2' \geq 0$, and $\mathrm{v}(\omega_1', \omega_2') = -1$ otherwise. Then, a recommended action $z^\bullet$ would belong to the interval $[z_L, z_H]$, where z_L and z_H are the points where the curves $CE_1(z)$ and $CE_2(z)$ respectively intersect the horizontal axis.

The upcoming sections will now make our construction more general and rigorous, and will show that policies generated in this manner share a number of desirable properties.

III. The Basic Framework

Consider an expert or model m which brings together some exogenous parameters $\tau \in \Upsilon$, policy (or control) variables $z \in Z$ and endogenous variables $x(z,\tau) \in X$. At each specific instances of τ and z, this model generates a scenario or forecast $\omega = m(x(z,\tau), z; \tau)$ which belongs to a set Ω. There is a total preorder over Ω, denoted $\lesssim$, which corresponds to the policymakers' appraisal of all scenarios: for any two scenarios ω and w in Ω, $\omega \lesssim$ w means that w is "better" than ω from the policymakers' stance.[14] Let the function $u : \Omega \rightarrow \mathbb{R}$ render this on a numerical scale, i.e. $\omega \lesssim \mathrm{w}$ if and only if $u(\omega) \leq u(\mathrm{w})$.

A. *Multiple-Scenario Assessments*

From now on, there will be $n > 1$ different models, denoted $m_1, ..., m_n$, drawn from a set M. At a given time, policymakers are then presented a variety of forecasts $\overline{\omega} = (\omega_1, \omega_2, ..., \omega_n)$ which belong to the cartesian product Ω^n; for $i = 1, ...n$, we have that $\omega_i = m_i(x_i(z,\tau_i), z; \tau_i)$, so all models feature the same policy variables (but not necessarily the same exogenous parameters, endogenous variables, or even relationships and structure linking variables and parameters). The preorder relation $\lesssim$ can be applied componentwise to obtain the canonical preorder $\preceq$ on Ω^n:[15]

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[14] Recall that a binary relation $\lesssim$ defined over the set Ω is a *total preorder* if, for all ω, w, $\mathrm{w}^\circ \in \Omega$, (i) either $\omega \lesssim$ w or w $\lesssim \omega$ (*completeness property*), (ii) $\omega \lesssim \omega$ (*reflexivity*), and (iii) $\omega \lesssim$ w and w $\lesssim \mathrm{w}^\circ$ implies $\omega \lesssim \mathrm{w}^\circ$ (*transitivity*). When $\omega \lesssim$ w and w $\lesssim \omega$, one usually writes w $\sim \omega$, meaning that w is "equivalent" to ω from the policymakers' viewpoint. When $\omega \lesssim$ w but not w $\lesssim \omega$, we write $\omega <$ w.

[15] A more general framework would have several sets Ω_i with respective complete preorder $\lesssim_i$, $i = 1, ..., n$ (meaning that the range of possible forecasts and their ranking may depend on who the underlying model or expert is), while the function u takes values in a completely preordered (not necessarily numerical) set. The results shown below are still valid under these extensions.

$$\overline{\omega} \preceq \overline{\mathrm{w}} \quad \text{if and only if} \quad \omega_i \lesssim \mathrm{w}_i \quad \text{for all } i = 1, ..., n \ .$$

If $\omega_i < \mathrm{w}_i$ for all $i = 1, ..., n$, we write $\overline{\omega} \prec \overline{\mathrm{w}}$. One can also construct the *assessment function* $U : \Omega^n \to \mathbb{R}^n$ as $U(\overline{\omega}) = (u(\omega_1), ..., u(\omega_n)) = (u_1, ..., u_n)$. Let $\Sigma = U(\Omega^n) \subseteq \mathbb{R}^n$ denote the image of U; the function $U : \Omega^n \to \Sigma$ is then *surjective*, by definition. We will be using this fact shortly.

B. *Policy Rules*

Without loss of generality, the number 0 will be seen as a threshold or *target* for policy. Let $\Sigma_- = \Sigma \setminus \mathbb{R}^n_+ = \{\overline{u} = (u_1, ..., u_n) \in \Sigma : u_i < 0 \ \text{ for some } i\}$, supposing that Σ_- is nonempty and strictly included in Σ; each element of the set $\Omega^n_- = U^{-1}(\Sigma_-)$ thus contains at least one scenario policymakers deem bad enough to warrant some remedial action.

Assume that a single action z'(which may itself involve the simultaneous or sequential deployment of several policy instruments) is undertaken at a given time, and that each expert or model i is able in this case to provide a revised scenario $\omega'_i = m_i(x_i(z', \tau_i), z'; \tau_i)$. Policy intervention can then be portrayed as a function $\Phi : \Omega^n \to \Omega^n$ such that $\Phi(\overline{\omega}) = \overline{\omega}'$ captures its impact (according to the same n models) $\overline{\omega}' = (\omega'_1, \omega'_2, ..., \omega'_n)$ on all the initial scenarios $(\omega_1, \omega_2, ..., \omega_n)$ comprised in $\overline{\omega}$. In what follows, we refer to Φ as a *policy rule.*[16]

C. *Policy Evaluation*

Modified scenarios and forecasts are ultimately subject to overall appraisals. These are given by the function v: $\Omega^n \to Q$, where Q is a set of real numbers. Below, we denote Q_+ the intersection $Q \cap \mathbb{R}_+$, and we assume that Q_+ is a nonempty strict subset of Q.

[16] This is an "implicit" representation of a policy rule, so to speak. An explicit, and perhaps more standard, description would have $z' = z(\tau_1, ..., \tau_n; \Phi)$.

In their account of monetary policy, Levin and Williams (2003, p. 946) report that a policymaking committee usually seeks policy outcomes that are acceptable to all its members. In agreement with this stylized fact, the function v can be supposed to meet the following assumption.

ASSUMPTION 1 (*Unanimity*). $\overline{\omega} \in \Omega^n_- \Leftrightarrow \mathrm{v}(\overline{\omega}) \leq 0$.

In other words, policies that perform very poorly in at least one of the committee members' model, and thus fail to be consensual, will receive a nonpositive score. Let $G : \Omega^n \to Q$ denote the composition $G = \mathrm{v} \circ \Phi$ of the functions v and Φ. Under Assumption 1, it can be understood as expressing the policymakers' value of (or ex post willingness-to-pay for) the scenario modifications implied by the policy rule Φ. Accordingly, the set Q, with generic element q, can be seen as a set of quotes.

This completes the background necessary to lay out our approach.

IV. The General Method

The foundation of our approach is the following adaptation to the present context and notation of a theorem demonstrated in Craven (1972; theorem 2.1). This theorem is a nonlinear generalization of the well-known *Farkas's Lemma* of convex analysis.

THEOREM: *If* $U : \Omega^n \to \Sigma$ *is surjective, then*

$$U(\overline{\omega}) = U(\overline{\mathrm{w}}) \Rightarrow G(\overline{\omega}) = G(\overline{\mathrm{w}}) \text{ and} \tag{2}$$

$$U(\overline{\omega}) \in \Sigma_- \Rightarrow G(\overline{\omega}) \in Q_+ \tag{3}$$

for all $\overline{\omega}, \overline{\mathrm{w}} \in \Omega^n$ *if and only if there exists a function* $\pi : \Sigma \to Q$ *such that*

$$G = \pi \circ U \quad \text{and} \quad \pi(\Sigma_-) \subset Q_+ \ . \tag{4}$$

The above framework ensures that the theorem's hypothesis is satisfied.[17] A policy rule Φ that fulfills condition (3) can be called *effective*; it amends any combination of bad scenarios so that no further intervention is needed. Condition (2) is one of *consistency*: scenarios which get the same rankings trigger equivalent policies (from the policymakers' standpoint). Of course, one may have $\Phi(\overline{\omega}) \neq \Phi(\overline{\mathrm{w}})$ but $G(\overline{\omega}) = G(\overline{\mathrm{w}})$, so this condition does not exclude applying different treatments to similar scenarios (as the above example illustrates). Also, condition (2) does not apply to situations where $\overline{\mathrm{w}}$ is a permutation of $\overline{\omega}$, for in this case $U(\overline{\omega}) \neq U(\overline{\mathrm{w}})$ most of the time; the identity of an expert who supports a given scenario may thus matter for policy.

Since $\pi(\Sigma_-) \subset Q_+$, so $\pi(u_1, ..., u_n)$ is positive if an initial assessment u_i is bad ($u_i < 0$ for some i), the "dual" function π can be typically interpreted as indicating the "price" policymakers would pay to avoid an original set of potential welfare levels $\{u_1, ..., u_n\}$. The theorem then says that a consistent and effective policy rule must be such that the policymakers' collective *valuation of its projected impact* $G = \mathrm{v} \circ \Phi$ matches their joint *willingness-to-pay* $\pi \circ U$ to escape the initial forecasts. The proof of this statement follows.

Proof (Craven 1972): Suppose that conditions (2) and (3) are true. Then, for each $\overline{u} \in \Sigma$, let $\pi(\overline{u}) = G(\overline{\omega})$, where $\overline{\omega}$ is any element of Ω^n such that $U(\overline{\omega}) = \overline{u}$. Condition (2) ensures that π is a well-defined function. Furthermore, its domain is Σ, since $U(\Omega^n) = \Sigma$, and $G = \pi \circ U$ by definition. If $\overline{u} \in \Sigma_-$, then $\overline{u} = U(\overline{\omega})$ for some $\overline{\omega} \in \Omega^n_-$, and (3) entails

$$U(\overline{\omega}) \in \Sigma_- \Rightarrow G(\overline{\omega}) = \pi(\overline{u}) \in Q_+ \ ,$$

[17] If Ω^n, Σ and Q are topological spaces, U is a continuous open map and G is continuous, one can also show that the price schedule π must be continuous (see Craven 1972).

so $\pi(\Sigma_-) \subset Q_+$. Conversely, let $\pi : \Sigma \to Q$ satisfy (4); the function G defined as $G = \pi \circ U$ obviously meets (2) and (3). ■

This theorem justifies seeking a suitable policy Φ by solving the equation

$$\mathrm{v} \circ \Phi = \pi \circ U \quad . \tag{1}$$

Some remarks are in order at this point:

• The construction first relies on the function U, which assigns a score to each scenario and should naturally be part of any rule-based policy. Scores may belong to a continuous or discrete scale, the former meaning they are given, for instance, by some risk measure, the latter that a scenario should fit a finite number of categories (e.g., “good” or “bad”).

• The policy evaluation v and willingness-to-pay π are also essential. They may be elicited from policymakers, either directly, through voting, auctioning or a survey, or indirectly, via the analysis of related choices, hedonic prices or experiments. In many cases, policy evaluation and the function v might actually be ordered by decree (see, e.g., Davies 2004).

• Once U, v and π are at hand, one can find Φ by working out equation (1) directly, as in the example of Section II, or by taking a quasi-inverse $\mathrm{v}^{[-1]}$ of v so that[18]

$$\Phi = \mathrm{v}^{[-1]} \circ \pi \circ U \quad . \tag{5}$$

• *Existence* of a proper action $z^\bullet$ is ensured, in particular, if the function $\mathrm{v} \circ \Phi \circ m(x(\cdot, \tau), \cdot; \tau)$ is continuous over the set of controls Z and spans the value $\pi \circ U(\overline{\omega})$.[19]

[18]The mapping $\mathrm{v}^{[-1]} : Q \to \Omega^n$ is a quasi-inverse of v if $\mathrm{v} \circ \mathrm{v}^{[-1]} \circ \mathrm{v} = \mathrm{v}$. Every function has a quasi-inverse (if the Axiom of Choice holds). Yet, $\mathrm{v}^{[-1]}$ is not unique unless v is a bijection. Note that $\mathrm{v}^{[-1]}$ can be a quasi-inverse of v but not vice versa; this fact must be dealt with in order to use (5).

[19]This assertion uses the following general version of the *intermediate-value theorem*, which is a specialized-to-our-context version of the one stated for instance in Munkres (2000): “Let Z be a connected space and $J = \mathrm{v} \circ \Phi \circ m \circ x(\cdot, \tau)$ be a continuous function from Z to Q. If there are z_1 and z_2 in Z such that $J(z_1) < \pi \circ U(\overline{\omega}) < J(z_2)$, then there exists a $z^\bullet$ in Z such that $J(z^\bullet) = \pi \circ U(\overline{\omega})$.”

To strengthen the present role and interpretation of π, let us replace the theorem's condition that $\pi(\Sigma_-) \subset Q_+$ with the following stronger requirement.

ASSUMPTION 2 (*Strict willingness-to-pay*). $\overline{u} \in \Sigma_- \Leftrightarrow \pi(\overline{u}) > 0$.

As we shall now see, policy rules built using a willingness-to-pay that satisfy the latter will have appealing characteristics.

V. Some Key Normative Properties

The literature on model uncertainty normally stipulates *a priori* that the designed policy rules possess certain desirable properties. One such property is *robustness*, which calls for policies that may not be optimal under some models but will be acceptable if any of the *ex post* scenarios materializes (see, e.g., Hansen and Sargent 2008). Another one is *simpleness*, which precludes policies from fine-tuning the available models to achieve specific scenarios. This section shows that our approach actually *endows* the obtained policy rules with these features, and with other valuable attributes of decision making.

A. *Process Attributes*

One first pleasing attribute of a policy rule Φ which solves equation (1) under Assumptions (1) and (2) is that it eliminates all the bad initial scenarios and never induces an unfavorable one. Hence, when a model i initially renders a forecast ω_i such that $u(\omega_i) < 0$, nobody would challenge the rule.

PROPERTY 1 (*Consensual remedy*): For all $\overline{\omega} \in \Omega_-^n$, $\Phi(\overline{\omega}) \notin \Omega_-^n$.

PROOF: Suppose there exists some $\overline{\omega} \in \Omega_-^n$ with $\Phi(\overline{\omega}) \in \Omega_-^n$. By Assumption 1, we must have that $\mathrm{v} \circ \Phi(\overline{\omega}) \leq 0$. However, since $\overline{\omega} \in \Omega_-^n$, $U(\overline{\omega}) \in \Sigma_-$ and $\pi \circ U(\overline{\omega}) > 0$ by Assumption 2. This contradicts the fact that $\mathrm{v} \circ \Phi(\overline{\omega}) = \pi \circ U(\overline{\omega})$. ■

By contrast, policy intervention will not receive unanimous support when all initial scenarios are good, for it will give rise to at least one bad forecast.

PROPERTY 2 (*Self-restraint*): Let $\Omega_+^n = \Omega^n \setminus \Omega_-^n$. For all $\overline{\omega} \in \Omega_+^n$, $\Phi(\overline{\omega}) \notin \Omega_+^n$.

PROOF: Assume there exists some $\overline{\omega} \in \Omega_+^n$ with $\Phi(\overline{\omega}) \in \Omega_+^n$. By Assumption 1, we must have that $\mathrm{v} \circ \Phi(\overline{\omega}) > 0$. However, since $\overline{\omega} \in \Omega_+^n$, $U(\overline{\omega}) \in \Sigma \setminus \Sigma_-$ and $\pi \circ U(\overline{\omega}) \leq 0$ by Assumption 2. This contradicts the fact that $\mathrm{v} \circ \Phi(\overline{\omega}) = \pi \circ U(\overline{\omega})$. ■

A direct consequence of these properties is that Φ does not have a fixed point. This means that no policy intervention is without consequences on the *ex post* scenarios.

PROPERTY 3 (*Non neutrality*): For all $\overline{\omega} \in \Omega^n$, $\Phi(\overline{\omega}) \neq \overline{\omega}$.

This third property may serve as a warning on policymakers to use the policy rule wisely. It may alternatively be viewed as a rough safeguard against indifferent or stubborn experts who would maintain their initial forecast after the policy rule was applied.

Finally, call an application $\Gamma : \Omega^n \rightarrow \Omega^n$ *decomposable* if there are functions $\gamma_i : \Omega \rightarrow \Omega$, $i = 1, ..., n$, such that $\Gamma(\overline{\omega}) = (\gamma_1(\omega_1), ..., \gamma_n(\omega_n))$ for all $\overline{\omega} = (\omega_1, ..., \omega_n) \in \Omega^n$.[20] A policy rule Φ constructed as above will not have this feature.

PROPERTY 4 (*Holism*): The policy rule $\Phi : \Omega^n \rightarrow \Omega^n$ is not decomposable.

PROOF: Suppose instead that $\Phi(\overline{\omega}) = (\varphi_1(\omega_1), ..., \varphi_n(\omega_n))$ for all $\overline{\omega} = (\omega_1, ..., \omega_n) \in \Omega^n$. Take now some $\overline{\omega}^\diamond = (\omega_1^\diamond, ..., \omega_n^\diamond) \in \Omega_+^n$ so that $u(\varphi_1(\omega_1^\diamond)) < 0$, and consider an n-tuple

[20] This is a stronger form of decomposability. In mathematics and computer science, the decomposition of a multivalued function $\Gamma : \Omega^n \rightarrow \Omega^n$ involves some functions $\gamma_1, ..., \gamma_n : \Omega^n \rightarrow \Omega$ and $\Lambda : \Omega^n \rightarrow \Omega^n$ such that $\Gamma(\overline{\omega}) = \Lambda(\gamma_1(\overline{\omega}), ..., \gamma_n(\overline{\omega}))$ for all $\overline{\omega} \in \Omega^n$.

$\overline{\omega}^{\nabla} = (\omega_1^{\diamond}, \omega_2, ..., \omega_n)$ where $u(\omega_n) < 0$. We then have that $\Phi(\overline{\omega}^{\nabla}) = (\varphi_1(\omega_1^{\diamond}), ..., \varphi_n(\omega_n))$ with $\varphi_1(\omega_1^{\diamond}) < 0$, which contradicts Property 1. ■

In concrete terms, Property 4 says that the way a policy intervention determined by Φ is going to amend an original scenario will depend on all the scenarios initially submitted to policymakers.[21] This calls attention to the effect an upstream decision (which could be based on strategic, ideological or epistemological considerations) to let a scenario in or not might have on the design of policy. We shall come back on this in the conclusion.

B. *Robustness*

If one is ready to assume that the set Ω^n, partially ordered by $\preceq$, is a complete lattice,[22] Property 3 combined with some fixed-point theorems of lattice theory (see Brian Davey and Hilary Priestley 2002, theorems 8.22 and 8.23) implies that the policy rule Φ is neither order-preserving (or monotone) nor *all-improving* - the latter meaning that $\overline{\omega} \prec \Phi(\overline{\omega})$ for all $\overline{\omega} \in \Omega^n$. This characteristic actually holds on the very domain Ω^n_- where policy intervention is needed.

Property 5 (*Imperfect enhancement*): For at least one $\overline{\omega} \in \Omega^n_-$, we have that $\overline{\omega} \nprec \Phi(\overline{\omega})$.

Proof: Suppose instead that $\overline{\omega} \prec \Phi(\overline{\omega})$ for all $\overline{\omega} \in \Omega^n_-$. Let

$$\Omega^{\triangleright}_- = \{\overline{\omega} = (\omega_1, ..., \omega_n) \in \Omega^n \mid u(\omega_i) = u_i < 0 \text{ for all } i \neq 1\}.$$

[21] This is actually true as well for policies that suit the maximin criterion.

[22] Recall that $(\Omega^n, \preceq)$ is a *complete lattice* if, in addition to properties (ii) and (iii) listed in footnote 10, we have that (iv) for all $\overline{\omega}$, $\overline{\mathrm{w}} \in \Omega^n$, $\overline{\omega} \preceq \overline{\mathrm{w}}$ and $\overline{\mathrm{w}} \preceq \overline{\omega}$ implies $\overline{\omega} = \overline{\mathrm{w}}$ (*antisymmetry*) and (v) every subset of Ω^n has a least upper bound (supremum) and a greatest lower bound (infimum) in Ω^n (*completeness*). Property (iv), which makes $\preceq$ an order relation, forbids that two scenarios $\overline{\omega}$ and $\overline{\mathrm{w}}$ be equivalent without being identical (i.e. such that $\overline{\omega}_i \sim \overline{\mathrm{w}}_i$ for all i); to satisfy this, one may take Ω as a set made of collections of equivalent scenarios, each collection being represented by one of its elements.

Since Ω^n is a complete lattice, the set $\Omega_-^{\triangleright}$ has a supremum $\vee\Omega_-^{\triangleright} = \overline{\omega}^{\triangleright} = (\omega_1^{\triangleright}, ..., \omega_n^{\triangleright})$. Clearly, $u(\omega_i^{\triangleright}) = u_i < 0$ for all $i \neq 1$, so $\overline{\omega}^{\triangleright} \in \Omega_-^{\triangleright}$. Taking $\Phi(\overline{\omega}^{\triangleright})$, consider now the n-tuple $\overline{\omega}^{\triangle} = (\Phi_1(\overline{\omega}^{\triangleright}), \omega_2^{\triangleright}, ..., \omega_n^{\triangleright})$ which differs from $\overline{\omega}^{\triangleright}$ in having the first component of the latter replaced by the first component $\Phi_1(\overline{\omega}^{\triangleright})$ of $\Phi(\overline{\omega}^{\triangleright})$. Such a n-tuple also belongs to $\Omega_-^{\triangleright}$, so we must have that $\Phi_1(\overline{\omega}^{\triangle}) \lesssim \omega_1^{\triangleright}$. This inequality contradicts our initial assumption. ■

This property could be observed in the example of Figure 2, where we had $u(\omega_2') = a_2 - z^{\bullet} - \alpha\frac{(1-z^{\bullet})\sigma_2^2}{2} < a_2 - \alpha\frac{\sigma_2^2}{2} = u(\omega_2)$. Together with Property 1, it captures the meaning of *robustness*: the policy rule Φ fulfills its objectives in taking care of the unwelcome original scenarios, sometimes at the expense of the good ones (hence in a nonoptimal way with respect to some models), but never to the point of changing the latter into bad ones.

Properties 1 and 5 suggest in addition that policies satisfying equation (1) are indeed *precautionary*.[23] Reporting on the Federal Reserve Chairman's conference to the 2004 annual meeting of the American Economic Association, Walsh (2004) defined a precautionary policy as one that "would err on the side of reducing the chance that the more costly outcome occurs." The maximin criterion was then seen as a practical way to bring about such a policy. Our approach now offers a distinct alternative, which also gives priority, but not exclusive attention, to the worst cases.

C. *Simpleness*

Simple policy rules were advocated decades ago by Friedman (1968), considering the complexity of the economy and the ensuing uncertainty of policymakers. In the present

[23] See Barrieu and Sinclair-Desgagné (2006) for further discussion on this point and the related implementation of the so-called *Precautionary Principle*.

context, this requirement can be understood as saying that the range of working policies $Z(\varpi) \subset Z$ should be narrower (thereby forcing policy rules to be less elaborate), when the number of disagreeing scenarios comprised in ϖ increases.[24] Our approach will obey this desideratum in at least two occasions.

Property 6, case I (*Decreasing policy range*): Let $\mathrm{v}(\overline{\mathrm{w}}) = -1$ if $u(\mathrm{w}_i) < 0$ for at least one i, and $\mathrm{v}(\overline{\mathrm{w}}) = 1$ otherwise. Starting with at least one bad scenario, the set of policies for which equation (1) is satisfied decreases with the number of scenarios n.

Proof: Take $\varpi = (m_1(x_1(z,\tau_1), z; \tau_1), ..., m_n(x_n(z,\tau_n), z; \tau_n))$ in Ω_-^n, and denote

$$Z(\varpi) = \{z' \mid \mathrm{v}(m_1(x_1(z',\tau_1), z'; \tau_1), ..., m_n(x_n(z',\tau_n), z'; \tau_n)) = 1\}$$

the set of policies that can then solve equation (1). Consider the augmented family of scenarios (ϖ, ω_{n+1}) where $\omega_{n+1} = m_{n+1}(x_{n+1}(z,\tau_{n+1}), z; \tau_{n+1})$. This configuration belongs to Ω_-^{n+1} by definition, and the set of successful policies, which is (abusing notation)

$$Z(\varpi, \omega_{n+1}) = \{z' \mid \mathrm{v}(m_1, ..., m_n(x_n(z',\tau_n), z'; \tau_n), m_{n+1}(x_{n+1}(z',\tau_{n+1}), z'; \tau_{n+1})) = 1\} \ ,$$

must be a subset of $Z(\varpi)$. ■

In other words, when *ex post* policy appraisals take only two values, as will happen if they express collective decisions to endorse ($\mathrm{v}(\overline{w}) = 1$) or disapprove ($\mathrm{v}(\overline{w}) = -1$) all modified configurations of scenarios, greater model uncertainty in circumstances where policy intervention is warranted (according to properties 1 and 2) will reduce the policymakers' options and so make fine-tuned remedies less likely.

To introduce the second case, let

$$Z_{q,n}(\varpi) = \{z' \mid \mathrm{v}(m_1(x_1(z',\tau_1), z'; \tau_1), ..., m_n(x_n(z',\tau_n), z'; \tau_n)) = q\}$$

[24]In a recent paper, Al-Najar and Pai (2009) adopt a similar view.

be the set of successful actions if $\pi \circ U(\overline{\omega}) = q$. A similar conclusion now holds.

Property 6, case II (*Decreasing policy range*): Suppose that (i) $Z_{q',n}(\overline{\omega}) \subset Z_{q,n}(\overline{\omega})$ when $0 < q < q'$, for all n and $\overline{\omega}$, (ii) $Z_{q,n+1}(\overline{\omega}, \omega_{n+1}) \subset Z_{q,n}(\overline{\omega})$ for all n, q, $(\overline{\omega}, \omega_{n+1})$, and (iii) π increases with the *relative* number of bad scenarios. Then $Z(\overline{\omega}, \omega_{n+1}) \subset Z(\overline{\omega})$ when $\overline{\omega} \in \Omega^n_-$ and $u(\omega_{n+1}) < 0$.

Proof: Take $(\overline{\omega}, \omega_{n+1})$ such that $\overline{\omega} \in \Omega^n_-$ and $u(\omega_{n+1}) < 0$. Let $\pi \circ U(\overline{\omega}) = q$ and $\pi \circ U(\overline{\omega}, \omega_{n+1}) = q'$. By (iii), we have that $0 < q < q'$. It now follows that

$$Z(\overline{\omega}, \omega_{n+1}) = Z_{q',n+1}(\overline{\omega}, \omega_{n+1}) \subset Z_{q,n+1}(\overline{\omega}, \omega_{n+1}) \subset Z_{q,n}(\overline{\omega}) = Z(\overline{\omega}) \ ,$$

where the first and second inclusions come respectively from (i) and (ii). ■

In other words, the following two conditions suffice again to guarantee that greater model uncertainty will induce simpler policy rules in circumstances where policy intervention is increasingly justified ($u(\omega_{n+1}) < 0$): first, having to meet a higher willingness to pay (assumption i) or deal with more disagreeing experts (assumption ii) must reduce the range of remedies, second, the policymakers' collective quote to avoid an initial situation must go up as the proportion of bad scenarios is larger (assumption iii).

VI. Two Potential Applications

This section will now briefly sketch our approach's potential contribution to two significant policy debates. The first one concerns banking regulation and the determination of capital requirements. There is currently no consensual view about the latter. One suggestion is therefore to be *eclectic* on the matter, drawing from various risk formulae and assessments to set appropriate capital reserves. Our approach may provide a useful framework to do so. The second one relates to climate policy and the validity of cost-benefit analysis in this context. As pointed out by Weitzman (2007), it is largely the low-probability high-impact consequences of climate change which are uncertain; this brings about fat-tailed predictive distributions which may force policymakers to give up on cost-benefit analysis based on discounted expected utility. Turning then to this paper's approach would still guarantee that the chosen policies retain key normative properties.

A. *Banking Regulation*

Banks and other financial institutions are generally required to hold minimal capital levels to protect deposits against a potential drop in the value of their assets. It is widely accepted, however, that capital reserves should vary according to a bank's risk exposure: one that heavily invested in highly liquid and very safe securities (such as U.S. government bonds), for example, should not need to keep the same amount of reserves. Under the 2006 Basel II agreement, regulators can thus allow an institution to use credit ratings from certain approved agencies when calculating its net capital reserve requirements. Those agencies might have based their ratings on the so-called "value-at-risk" (VaR), which measures the risk of loss in a portfolio of financial assets (see, e.g., Printsker 1997). More recently, some market measurements, based upon traded instruments such as credit default swaps (Hart and Zingales 2009) or the difference between the Libor rate and the overnight swap rate (Taylor and Williams 2009), have been proposed as alternatives or complements to achieve the same goal.

All these schemes have merits and defects. The price of credit default swaps (CDS), for instance, reflects in principle the probability (from the market's perspective) a given institution is insolvent, but the CDS market is believed by many to be rather thin and subject to distortions. Meanwhile, significant research is going into allowing the VaR to cover catastrophic outcomes and meet other desiderata (see, e.g., Rockafellar and Uryasev 2002; Elsinger at al. 2006).

A regulator who then (not unreasonably) chooses to be eclectic on the matter could use our framework as follows.[25] In this paper's language, take each "valuable" (according to policy makers) source of warnings about a given financial institution - rating agencies, credit default swaps, etc. - as if it were a particular "model" m_i, and its corresponding ratings or prices at a given time as a "scenario" ω_i. Scenarios from any model clearly depend on the same policy variable: the institution's current capital reserves. Policy triggers are now captured by the functions $u_i(\omega_i)$; to render Hart and Zingales (2009, p. 14)'s suggestion, for example, if ω_i stands for CDS prices over the last 30 trading days, then $u_i(\omega_i)$ might be negative when those prices were above some pre-specified threshold for at least 20 days. Let $\pi(\ldots\ u_i(\omega_i)\ \ldots)$ indicate the policymakers' joint apprehension concerning the institution's financial health and its (possibly systemic) consequences, based on the available scenarios and triggers (for instance, $\pi(\ldots\ u_i(\omega_i)\ \ldots) = 1$ whenever $u_i(\omega_i) < 0$ for

[25] Eclecticism seems indeed reasonable here, considering the following account (from *The Economist*, "Base camp Basel," January 23^{rd} 2010, p. 66-68): "The Basel regime (European and American banks use either version 1 or 2) represents a monumental, decades-long effort at perfection, with minimum capital requirements carefully calculated from detailed formulae. The answers were precisely wrong. Five days before its bankruptcy Lehman Brothers boasted a "Tier 1" capital ratio of 11%, almost three times the regulatory minimum." Of course, policymakers might as well pay less attention to seeking preventive trigger mechanisms and center instead on some capital insurance scheme to mitigate the costs of a crisis (as proposed by Kashyap et al. 2008).

at least one i, and $\pi(\ldots\ u_i(\omega_i)\ \ldots) = -1$ otherwise). With a similar criterion v for assessing *ex post* scenarios (for example, $\mathrm{v}(\ldots\ \omega_i'\ \ldots) = -1$ whenever $u_i(\omega_i') < 0$ for at least one i, and $\mathrm{v}(\ldots\ \omega_i'\ \ldots) = 1$ otherwise), capital requirements which are robust, consistent, holistic and effective (the latter ensuring that the financial institution is solvent with probability one) could finally be set by solving equation (1).

B. *Climate Policy*

As already mentioned in the introduction, climate change is one area where models and experts regularly disagree. Fuelling yet more discussions, Weitzman (2007, 2009b) recently argued that the current economic analyses of climate policy overly rely on assumptions that understate the inherent uncertainty surrounding low-probability disasters (about which, by definition, we have and will continue to have little data). Building on Geweke (2001)'s previous remarks, Weitzman (2009b) demonstrates what he terms the "Dismal Theorem," which roughly means that addressing such uncertainty the Bayesian way, thereby taking expectations of expectations or probability distributions of probability distributions, must yield fat-tailed posterior-predictive distributions (i.e., distributions which moment-generating function is infinite).[26] The upshot is that the existence of well-behaved (bounded, differentiable, etc.) expected utility, hence the possibility of crafting policy using standard cost-benefit analysis (CBA), will depend crucially on a modeller's oftentimes subjective choices of probability distributions and utility function.

[26] Recall, for example, that the posterior-predictive distribution of a normally distributed random variable with unknown mean and standard deviation (a thin-tailed distribution) is a Student-t distribution (a fat-tailed distribution) with degrees of freedom equal to the number of available observations minus 1.

In his rejoinder to Nordhaus (2009)'s comment on the Dismal Theorem, Weitzman (2009a) concedes that: "Fat tails and the implied limitations that prevent CBA from reaching robust conclusions are frustrating for economists. (...) What are we supposed to advise policy makers and politicians quantitatively about how much effort to spend on averting climate change if conclusions from modeling fat-tailed uncertainties are not clear-cut?" He then makes the following proposal:

> Some sort of tricky balance is required between being overwhelmed by fat-tailed logic into a Hamlet-like paralysis that leads to abandoning CBA altogether, and being underwhelmed into insisting that it is just another empirical issue to be sorted out by business-as-usual CBA. (...) In my opinion, economists need to emphasize more openly to the policy makers, the politicians, and the public that, while formal climate-change CBA may be helpful, there is a danger of possible overconfidence from undue reliance on subjective judgements about the probabilities and welfare impacts of extreme events. What we can do constructively as economists is to explain better the magnitudes of the unprecedent structural uncertainties that are involved, explain why this feature limits what we can say, and present the best CBAs and the most honest sensitivity analyses that we can under fat-tailed circumstances, including many different functional forms for extremes. At the end of the day, policy makers must decide what to do on the basis of admittedly sketchy economic analyses of a gray area that just cannot render clear robust answers.

This suggestion fits actually well with our framework. Let us view the supplied ω_i's as the climate scenarios that policy makers deem correct (after listening to an exhaustive panel of experts and becoming aware of the full range of explanations, limitations and sensitivity analyses, as the statement recommends). In the simplest mode, the assessments $u_i(\omega_i)$ may next be red or green flags; red if ω_i exhibits a very long and thick left-tail or is thin-tailed but has a low expected utility, and green otherwise. Suppose finally the "willingness-to-pay" $\pi(\ldots\ u_i(\omega_i)\ \ldots) = 1$ if at least one $u_i(\omega_i)$ is red but $\pi(\ldots\ u_i(\omega_i)\ \ldots) = -1$ if not, while the "policy assessment" $\mathrm{v}(\ldots\ \omega_i'\ \ldots) = -1$ unless all $u_i(\omega_i')$'s are green and v(... ω_i' ...) = 1 otherwise. Solving equation (1) then ensures that the obtained policies will be precautionary (as Weitzman 2009b says the Dismal Theorem is calling for, but in the precise sense discussed in subsection V.B above) and effective at preventing catastrophes.[27] Dealing with fat-tailed distributions (without a priori discarding them) does not mean, therefore, all direction will be lost in policy making.

VII. Concluding remarks

> Limited to our human terms and devices,
> we grasp the world variously. I think of
> the disparate ways of getting at the diameter
> of an impenetrable sphere: we may pinion
> the sphere in calipers or we may girdle it
> with a tape neasure and divide by π, but
> there is no getting inside.
>
> - W. C. Quine (1990) -

This paper's objective was to offer a general approach to design trigger policies under model uncertainty. Three ingredients were shown to be essential: the ability to rank and label ex ante scenarios (captured by the function U), an indication of the policymakers' willingness to avoid those scenarios (given by π), and some means to appraise ex post policy outcomes (provided by v). We argued such items should exist in any operational policy process (through mandatory cost-benefit analysis or voting procedures, for instance). An effective and consistent policy rule Φ may then be obtained by solving the equation

$$\mathrm{v} \circ \Phi = \pi \circ U \quad . \tag{1}$$

Under unanimous ex post appraisal and strict ex ante willingness-to-pay, such a policy rule was shown to have key properties like self-restraint, robustness and simpleness.

[27] According to Weitzman (2009b), p. 12: "(...) [The Dismal Theorem] embodies a very strong form of a 'generalized precautionary principle' for situations of potentially unlimited downside exposure." The article, however, contains no precise statement of such principle.

This scheme does not require knowing a representative policymaker's probabilitistic beliefs and utility function. It also remains applicable whether model uncertainty is due to empirical limitations or conflicting paradigms. Some practical matters still need to be dealt with, of course. First, solving equation (1) in general is likely to be a nontrivial task. To the usual criteria for model selection (see, e.g., Hendry and Mizon 2000), one may therefore add the relative ease in computing Φ. Second, keeping in mind the Lucas critique, one needs to understand how political and strategic factors could distort the observed assessments and declared willingness-to-pay. Given its influence on policy design, the set of relevant models might also be manipulated by some interested parties. Handling these concerns satisfactorily will require extending the present team-theoretic context to a strategic multiple-player one.[28] Third, when the functions U, v, and π are not inherent to the policy mechanism, there should be some systematic way to get them. The willingness-to-pay π, for instance, might be directly elicited from policymakers, using for example some form of prediction market (Wolfers and Witzewitz 2004), or estimated thanks to some recent advances in computer simulation (Epstein and Axtell 1996). Last, one ought to analyze a dynamic version of the current scheme which allows models to evolve and policymakers to learn, something proponents of model averaging or the ambiguity criteria have already done (see, e.g., Epstein and Schneider 2007). A first step in this direction would be to consider what happens to the policy rule Φ when the set of scenarios Ω shrinks or expands. At some point, the true scenario might not even be among those supplied.

[28] The classical theory of economic policy has already been taken in this direction by Acocella and Di Bartolomeo (2006).

This case remains a puzzle for the Bayesian and ambiguity approaches, which rely on additive probability distributions. Our method might adequately come to terms with it because beliefs concerning whether at least one forecast can be trusted can be embedded in the shadow price π.

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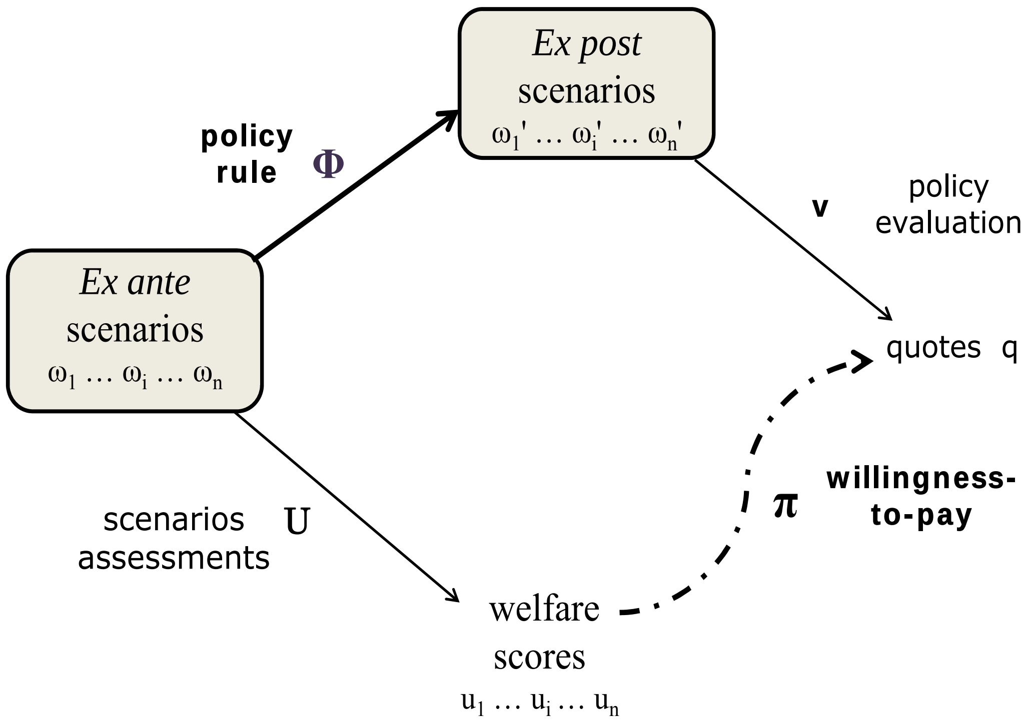

Figure 1. The basic construction

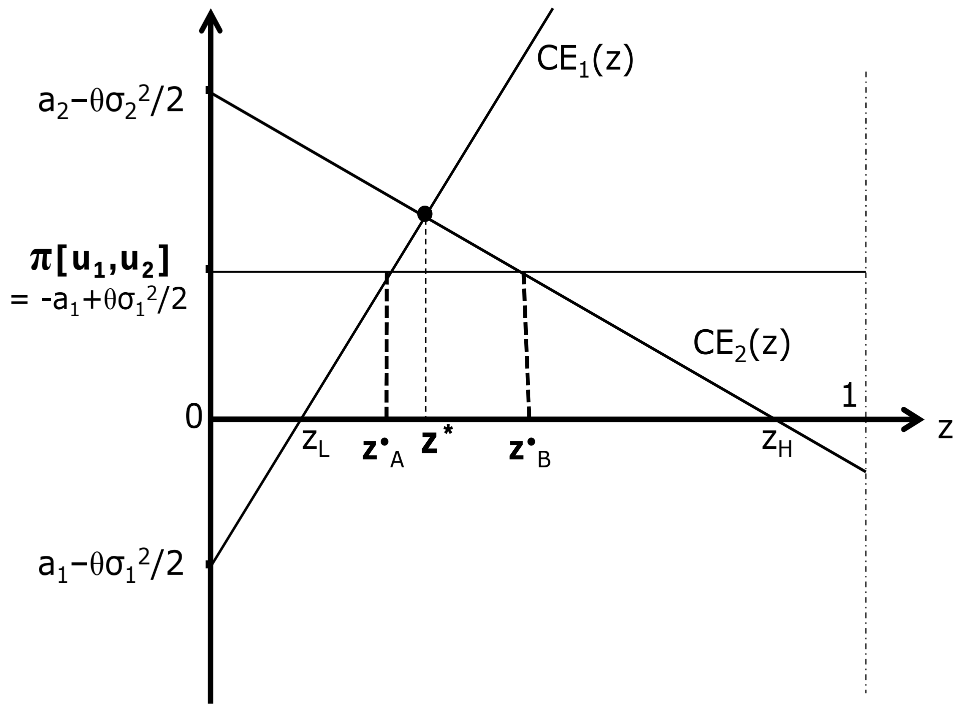

Figure 2. The maximin and this paper's solutions